THE STATE AND LOCAL GOVERNMENT BUDGET DECISION: AN APPLICATION OF THE LINEAR EXPENDITURE MODEL

By

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To Sumol, my Dearest Wife.

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Abstract of Dissertation Presented to the Graduate Council of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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This dissertation studies the state and local government budget decision by developing a model based on Stone-Geary's linear expenditure system. The model developed is different from the models of other studies in the same area in the following respects:

First, the model separates flat grants and matching grants making it possible to detect the effect of flat grants and matching grants on each category of expenditures and taxes. The separation of flat grants and matching grants is necessary for theoretical, econometrical and policy considerations.

Second, the model recognizes that expenditures, taxes and matching grants are determined simultaneously. Thus, the model avoids simultaneous bias in estimation, an error which has been common in most of the empirical studies in this area.

Third, the model uses relative prices as an allocation device. The use of price as an allocation device not only assigns an appropriate role to price but also makes it possible

to examine the effect of changes in realtive prices on expenditures and taxes.

Fourth, the expenditures are divided into categories by functions. This makes it possible to detect the own and cross effect of changes in prices, grants and other demographic variables on expenditure as a whole, and also expenditure categories by functions.

The model developed is estimated by using a full information maximum likelihood technique. The emperical result indicates that the model is statistically significant in its explanatory power. It is found, among other things that the effect of a reduction in federal taxes and the effect of a federal flat grant to the state and local government on state and local government expenditures are the same. The effects of these two actions on the state and local taxes are different, however. A change in the price of private goods as well as a change in the cost of police protections have very little effect on the expenditures of other state and local government goods. The change in the cost of education, however, has a considerable effect on state and local expenditures on other goods.

CHAPTER I

Introduction

This dissertation examines state and local government budget decisions. It does this by developing a model based on the linear expenditure system. This chapter states the purpose of the dissertation, introduces the model and outlines the study.

Most of the empirical studies of state and local government budget decisions are subject to criticism because of the ad hoc nature of the equations that were estimated. Studies which do derive the equations that were estimated have also been criticized either for not using the appropriate functional form of the utility function or for not using the appropriate estimation technique. The purpose of this dissertation is to (a) develop a model which is theoretically consistent and econometrically estimable and (b) estimate the model by using the appropriate estimation technique.

The model developed in this dissertation assumes a utility function of the state and local government budget decision makers. Maximization of this utility function subject to a budget constraint yields a system of linear expenditure equations. The system is, then, estimated by using a full-information maximum likelihood technique.

The model is different from other models in the same area in at least five respects:

First, it separates flat grants and matching grants making it possible to detect the effect of flat grants and matching grants on each category of expenditure and taxes. The separation of flat grants and matching grants is necessary for theoretical, econometric and policy reasons.

Second, the model recognizes that expenditures, taxes and matching grants are determined simultaneously. Thus, the madel avoids bias due to simultaneity, an error which has been common in most of the empirical studies of state and local government expenditures.

Third, the model incorporates relative prices or costs as influences on government budgetary choice. The use of prices has theoretical justification and also makes it possible to examine the effect of changes in relative prices on expenditures and taxes.

Fourth, the expenditures are divided into categories by functions. This makes it possible to detect the own and cross effects of changes in prices, grants and other demographic variables on expenditure as a whole, and also on several expenditure categories.

Fifth, the estimation technique is a full information maximum likelihood technique. This means that not only is the simultaneity in the variables recognized but the simultaneity in the parameters is recognized as well. The full information technique guarantees that the estimates of the parameters

are unique. In addition, the covariance matrix of the estimates of the parameters are provided. Moreover, a procedure for testing the significance of the overall model estimated by this particular technique is also provided.

Outline of Study

Chapter II evaluates the existing literature. Both theoretical and empirical studies are discussed. The contribution that each study made to the development of the theory of state and local government budget decision-making is described and evaluated. This evaluation reveals inadequacies of existing models and establishes the need for a new model which overcomes these inadequacies. In Chapter III, a new model is developed. Interpretation as well as some theoretical properties of the model are given. Chapter IV establishes the necessity of estimating price or cost equations. The price or cost equations are specified and estimated in this chapter.

Chapter V specifies and estimates grant equations. The reasons for the separation of flat and matching grants are given as well. Chapter VI describes the estimation techniques. All available techniques for the estimation of the linear expenditure system are evaluated. A comparison of the properties of the estimators and the stochastic specification of each technique is made. One of the procedures, i.e. Parks' procedure, is chosen to estimate the model. Chapter VII presents and interprets the empirical results and discussed policy implications. Chapter VIII suggests directions for future research in this area.

The Appendices include the derivation of the compensated demand function and a list of the data sources.

CHAPTER II

Previous Research on State and Local Government Budgetary Decisions

Existing studies of the state and local government budget decisions can be divided into two categories. One category emphasizes qualitative or theoretical analysis while the other examines the quantitative or empirical aspect of the budget decisions. These two categories are not mutually exclusive and involve considerable overlap. Studies without theoretical analysis are classified as theoretical whereas studies involving both theory and empirical work are classified as empirical studies.

2.1 Theoretical Studies

Theoretical studies take two approaches, namely, the individualistic approach and the organic approach.

The individualistic approach assumes that government does not exist as a separate economic entity. Therefore, the government does not possess a utility function of its own. Only individuals possess utility functions. The behavior of the government is the outcome of the interaction of the utility maximizing behavior of the citizen of that government, which is achieved through the political process. For example, if a majority rule is used the decision of the government can be expected to be identical with that of a median voter.

With these underlying assumptions, Bradford and Oates [1] demonstrate that the effect of giving flat grants to state and local governments is equivalent to a federal income tax reduction. No empirical studies have been really based on this approach. Borcherding and Deacon [2] try to base their empirical work on this approach but they are not successful. The main obstruction which prevents empirical studies from using this approach is that it relies heavily on unidentifiable individual behavior. For example, when majority rule is used, the behavior of the state and local government is identical to that of the median voter. The median voter is, however, not identifiable.

Borcherding and Deacon use per capita expenditure as a unit of observation. This, clearly, is not based on the individualistic approach. The per capita expenditure is a mean concept not a median concept. The use of a mean as a representative of the group is different from the use of a median. The mean involves all individuals and it does carry the idea of organic approach. The median implies an individualistic approach but the median voter is not identifiable.

The organic approach assumes that the government exists as a separate economic entity. The government has its own objective function. The objective function of the government may or may not have the individual's utility functions as arguments, but it has a unique property of its own. Under this approach Scott [3] analyses intergovernmental relations and generates various theories concerning flat and matching

grants. Most of the empirical studies are based on this approach.

2.2 Empirical Studies

The emirical literature on state and local government budget decisions is classified, in this chapter, according to the technique employed.

The nontheoretical single equation technique has the benefit of simplicity. A study using this technique may involve one type of expenditure, one type of tax revenue or any aggregation of revenues or expenditures. Demographic variables are used to explain a particular type of expenditure or taxes. In this technique taxes and expenditures are studied separately. In case of more than one expenditure category are examined in the same study, each type of expenditure is examined independently. This is clearly a partial equilibrium approach in the sense that only one particular expenditure being studied is allowed to vary while other expenditures are assumed to be constant. This technique gives biased estimates due to the failure to recognize the simultaneous nature of some variables. This technique also often gives rise to wrong conclusions because of the lack of theoretical derivation of the equation to be estimated. Examples of studies using this technique are Kurnow [4], and Ohls and Wales [5].

The nontheoretical simultaneous equation technique recognizes simultaneity explicitly. This technique uses a system of simultaneous equations to describe the behavior of state and local government revenue and expenditure. It

recognizes the interdependence between expenditures and revenues. Econometrically speaking, this technique is better than the single equation technique because simultaneous equation bias is avoided. Theoretically speaking, however, it is no better than the single equation approach. This is because both techniques give no solid theoretical grounds for the derivation of the systems of equations that are estimated. Since there exists no objective function in this type of studies, it appears as if the equations that are estimated are generated by the process of trial and error. Thus specification errors are quite likely. Horowitz [6] and O'Brien [7] are examples of the studies employing this technique.

In the single equation technique with theory, the equations to be estimated are derived by employing rigorous theoretical analysis. This technique, however, ignores the simultaneous nature of some variables. Certainly, the estimates of this technique are biased. An example of studies using this technique is Borcherding and Deacon [2].

The simultaneous equation technique with theory is the best technique that has been developed in the study of the state and local government budget decision. The assumption underlying this technique is that the state and local government revenue and expenditures are not independent and are in fact interrelated. The decision as to how much revenue is needed depends on the level of anticipated expenditure. This technique is distinct from the last one because in this

one the interdependency between expenditures, grants and taxes is explicitly recognized. The system of equations that are estimated is generated by the process of maximization of a utility function subject to a budget constraint. Henderson [8], Gramlich [9] and Gramlich and Galper [10] used this technique.

Henderson [8] uses a logex welfare function to generate a system of equations to explain local government expenditure and taxation. He postulates that the resultant public expenditure and tax decisions can be explained as if they were the result of maximizing a social welfare function subject to a social budget constraint. His welfare function is specified to be a function of the community's per capita income, per capita revenue from federal and state governments, population, per capita public expenditure and per capita private expenditure.

Gramlich [9] demonstrates that the utility approach and the budgetary process approach result in the same estimating equations. In the utility approach Gramlich uses a quadratic utility function. His utility function is a function of state and local expenditure, federal grant-in-aid, income, taxes, construction outlays and borrowing. Maximizing this utility function subject to a budget constraint gives rise to a system of two linear equations, namely, the expenditure and tax equation. The expenditure equation expresses expenditure as a linear function of grants and taxes. The tax equation expressed taxes as a linear function of income,

expenditures and grants. In the budgetary process approach, Gramlich writes two behavioral equations. He, then, shows that these two behavioral equations can be transformed to the same expenditure and tax equations as generated by the utility approach.

Gramlich and Galper [10] assume that the state and local government budgetary policy has four main objectives: (1) higher current expenditures; (2) higher private disposable income; (3) greater flows of services from the stock of tangible capital; (4) greater flows of services from the stock of net financial assets. They use the same functional form of the utility function as Gramlich [9]. The arguments in the utility function are the four goods and services mentioned above in real term. Maximizing the utility function with subject to a budget constraint, they obtained a system of nonlinear equation. The nonlinearity appears in both the variables and the parameters. They drop the nonlinear terms before they estimate the equations.

2.3 The Need for a New Model

Besides having an objective function, the models as used by Henderson, Gramlich and Gramlich and Galper also have the following common features.

1. The functional forms of the utility functions which they use are not familiar and economic interpretations cannot be given to their parameters. Neither the parameters of Gramlich's utility function nor the parameters of Gramlich and Galper's utility function are identifiable.

2. Goods and services are not separated into categories. Henderson puts all type of local expenditure in his G. Although Gramlich, and Gramlich and Galper have separated capital and current expenditures in their models, they do not have expenditure separated by functions. With this construction the effect of a change in taxes and grants on any particular goods cannot be detected.

3. None of the models has used price or cost as an allocation device. The prices as used by Gramlich and Galper are price indices. They are employed to transform monetary value into real value but they do not serve as an allocation device. In a utility maximization model, relative prices are needed to serve as guidelines for budget allocation.

4. None of the models has explicitly recognized the difference in the effect of flat grants and matching grants for each service category. The existing models cannot detect the effect of a change in the federal matching ratio for service k on the state and local government expenditure of service k and other services.

5. None of the studies has used a full information technique to estimate their parameters. The simultaneity in the variables are fairly well recognized. Recognition, however, has not adequately been given to the simultaneity in the parameters.

To understand the effect of a change in the price or cost of one service on the expenditure of that or other service and to measure the differences in the impact of a

change in flat grants and matching grants a new model is needed. In the new model, the estimating behavioral equations should be derived from a utility function whose parameters are identifiable and have economic interpretations. Differences in flat grants and matching grants should be recognized. Expenditure should be broken into categories by functions. Prices as allocation devices should be employed. Simultaneities in the variables as well as in the parameters should be recognized. In other words, a model which is not only powerful in policy analyzing but also theoretically consistent and empirically estimable with acceptable estimates is needed. The purpose of this dissertation is to provide such a model.

CHAPTER III

The State and Local Government Linear Expenditure-Revenue Model (SALGLERM)

In this chapter a model for the study of state and local government budget decisions is developed. This model is called the State and Local Government Linear Expenditure-Revenue Model (SALGLERM). The economic interpretation as well as some of theoretical properties of the model is discussed in this chapter.

3.1 Model Specification

In this study the assumption is made that state and local budget decision makers act as if they are maximizing the following utility function.

$$U = \sum_{k=1}^{n-1} a_k \ln(x_k - b_k) + a_n \ln\left(\frac{y_p - T}{p_n} - b_n\right) \qquad (3\text{-}1)$$

where

x_k = the amount of service k provided by the state and local government. This amount is in some unit of quantity which will be discussed later;

y_p = per capita income after federal taxes;

T = per capita state and local taxes;

p_n = price of private goods;

b_i, $i = 1, \ldots, n$ = minimum required quantity, a parameter to be estimated

a_i, $i = 1, \ldots, n$ = proportion of supernumerary income that allocated to good i, a parameter to be estimated.

The meaning of a_i and b_i will become clearer after the reduced form equations are derived.

$(y_p - T)$ represents per capita disposable income. $(\frac{y_p - T}{p_n})$ is the amount of private goods consumed by the individual in terms of some abstract unit.

The decision maker's utility function is a function of the state and local government services and the amount of private goods consumed by the individuals.

The parameters of the utility function are subject to three constraints.

$$x_k - b_k \geq 0, \quad k = 1, \ldots, n-1 \tag{3-2}$$

$$\frac{y_p - T}{p_n} - b_n \geq 0, \tag{3-3}$$

$$\sum_{k=1}^{n} a_k = 1, \quad 0 < a_k < 1 \tag{3-4}$$

Equation (3-2) and (3-3) state that the amount actually provided must be greater than or equal to the amount needed.

Equation (3-4) requires that each a_i must be between 0 and 1, and the summation of all a_i must be equal to 1. This requirement is for the simplicity of the computation. The a_i's are scale parameters. If there is no limitation on a_i, the mathematics will become more complicated but no more information is gained by allowing the sum of the a_i to vary.

The inclusion of $[a_n \ln (\frac{y_p - T}{p_n} - b_n)]$ in the utility function (3-1) guarantees that the taxing powers of the state and local governments have some limit. If more taxes

are collected and more of the x_k's are provided, less income will be left for private personal use. The decision maker has to choose the level of x_k's and T such that the utility is maximized subject to the budget constraint. In addition to the budget constraint, which will be discussed later, the constraints (3-2) and (3-3) guarantee that the minimum requirements are satisfied.

The decision maker's utility function is subject to the following budget constraint:

$$T + G = \sum_{k=1}^{n-1} (1 - \delta_k)\, p_k x_k \qquad (3\text{-}5)$$

G = per capita flat grant

δ_k, k = 1,..., n-1 = federal matching ratio for matching grants

p_k, k = 1,..., 6 = price or cost of provision of good k.

Equation (3-5) can be rewritten as

$$T + G + \sum_{k=1}^{n-1} \delta_k p_k x_k = \sum_{k=1}^{n-1} p_k x_k$$

This equation indicates that tax plus the flat and matching grants must be equal to government total expenditure. Borrowing does not appear in the budget constraint because only the current expenditure is dealt with and the assumption is made that no current expenditure is financed by borrowing.

The Lagrangian function can now be set up as follows:

$$L = \sum_{k=1}^{n-1} a_k \ln(x_k - b_k) + a_n \ln\left(\frac{y_p - T}{p_n} - b_n\right) + \lambda\left(T + G - \sum_{k=1}^{n-1} (1-\delta_k) p_k x_k\right) \qquad (3\text{-}6)$$

Differentiating with respect to x_k, T and λ yields,

$$\frac{\partial L}{\partial x_k} = \frac{a_k}{x_k - b_k} - \lambda(1 - \delta_k)p_k = 0, \quad k = 1,\ldots, n-1 \tag{3-7}$$

$$\frac{\partial L}{\partial T} = \frac{a_n}{\frac{y_p - T}{p_n} - b_n}\left(-\frac{1}{p_n}\right) + \lambda = 0 \tag{3-8}$$

$$\frac{\partial L}{\partial \lambda} = T + G - \sum_{k=1}^{n-1}(1 - \delta_k)\, p_k x_k = 0 \tag{3-9}$$

from (3-8)

$$\frac{-a_n}{y_p - T - b_n p_n} + \lambda = 0; \quad a_n = \lambda(y_p - T - p_n b_n) \tag{3-10}$$

from (3-7)

$$a_k = (1 - \delta_k)p_k(x_k - b_k), \quad k=1,\ldots, n-1 \tag{3-11}$$

Adding equations (3-10) and the n-1 equations of (3-11) yields,

$$\sum_{k=1}^{n} a_k = \lambda[(y_p - T - b_n p_n) + \sum_{k=1}^{n-1}(1 - \delta_k)p_k(x_k - b_k)] \tag{3-12}$$

using equation (3-4) yields,

$$1 = \lambda[(y_p - T - b_n p_n) + \sum_{k=1}^{n-1}(1 - \delta_k)p_k(x_k - b_k)] \tag{3-13}$$

From (3-9)

$$\sum_{k=1}^{n-1}(1 - \delta_k)p_k x_k - T = G \tag{3-14}$$

Substituting into (3-13)

$$\lambda = \frac{1}{y_p - b_n p_n + G - \sum_{k=1}^{n-1}(1 - \delta_k)p_k b_k} \tag{3-15}$$

From (3-11)

$$x_k = \frac{a_k + \lambda(1 - \delta_k)p_k b_k}{\lambda(1 - \delta_k)p_k}$$

$$x_k = \frac{a_k}{\lambda(1 - \delta_k)p_k} + b_k \qquad (3\text{-}16)$$

Substituting the value of λ from (3-15) into (3-16), we have

$$x_k = \frac{a_k[y_p + G - b_n p_n - \sum_{k=1}^{n-1}(1 - \delta_k)p_k b_k}{(1 - \delta_k)p_k} + b_k$$

or

$$x_k = \frac{a_k}{(1 - \delta_k)p_k}\left[y_p + G - b_n p_n - \sum_{k=1}^{n-1}(1 - \delta_k)p_k b_k\right] + b_k \qquad (3\text{-}17)$$

Equation (3-17) is the demand function for service k.

Multiplying equation (3-17) through by p_k, we have

$$p_k x_k = \frac{a_k}{1 - \delta_k}\left[y_p + G - b_n p_n - \sum_{k=1}^{n-1}(1 - \delta_k)p_k b_k\right] + p_k b_k$$

or

$$p_k x_k = p_k b_k + \frac{a_k}{1 - \delta_k}\left[y_p + G - b_n p_n - \sum_{k=1}^{n-1}(1 - \delta_k)p_k b_k\right] \qquad (3\text{-}18)$$

$$k=1,\ldots,n-1$$

from (3-10)

$$T = y_p - p_n b_n - \frac{a_n}{\lambda} \qquad (3\text{-}19)$$

Substituting the value of λ from (3-15), we have

$$T = y_p - p_n b_n - a_n\left[y_p - b_n p_n + G - \sum_{k=1}^{n-1}(1 - \delta_k)p_k b_k\right] \qquad (3\text{-}20)$$

Equations (3-18) and (3-20) are the reduced form equations to be estimated. They express each endogeneous variable as a function of exogeneous variables. Equation (3-17) which is the demand equation expresses the quantity of x_k demanded as a function of income, grants, price or cost of the provision

of service k and price or cost of the provision of other goods and services. Equation (3-18) is an expenditure equation of the k^{th} service. It expresses the amount of expenditure for service k as a function of per capita income, grants, the price or cost of the provision of service k and prices or costs of the provision of other goods and services.

Applying the dimension analysis[1] to equation (3-18), it is found that a is dimensionless and b has the same dimension as x. Then the expenditure equation (3-18) can be interpreted as follows:

$p_k x_k$ is the total current expenditure on service k. $p_k b_k$ represents the expenditure on the necessary level of service k. This can be called the *necessary expenditure*. The amount b_k is provided regardless of provision cost or price. The fact that b_k represent the amount provided regardless of price can be seen by recognizing that b_k appear as the intercept of the demand equation for good k (see equation (3-17)). $y_p + G$ is the spendable budget. Out of this budget, the decision maker can choose to spend either on x_k or other x's or allow private citizens to spend more on private goods.

$b_n p_n + \sum_{k=1}^{n-1} (1 - \delta_k) p_k b_k$ is the amount of money spent on the minimum requirements of all goods and services

$[y_p + G - (b_n p_n + \sum_{k=1}^{n-1} (1 - \delta_k) p_k b_k)]$ represents the amount of money left over from acquiring the minimum level of all

[1] For more information on the dimension analysis, see Theil [11], page 438.

goods and services. $a_k[y_p + G - b_n p_n - \sum_{k=1}^{n-1} (1 - \delta_k)p_k b_k]$ is the portion of income left over from acquiring the minimum level of all goods and services, which is allocated to service k. The a_k represents the proportional factor with which the leftover income is allocated. However, part of the expenditures are financed by the federal government according to the federal aid to state and local government programs. This is why $\frac{1}{1 - \delta_k}$ enters. If state and local governments allocate a_k of the leftover income to service k, $\frac{1}{1 - \delta_k}$ of the leftover income will be the actual expenditure for this portion. The portion $\frac{1}{1 - \delta_k}[y_p + G - b_n p_n - \sum_{k=1}^{n-1} (1-\delta_k)p_k b_k]$ can be termed the *alterable expenditure*.

In short, the expenditure function states that the total current expenditure on service k is equal to the *necessary expenditure* plus the *alterable expenditure*.

Equation (3-20) is the tax equation and can be interpreted as follows:

y_p = per capita income after federal tax;

$p_n b_n$ = necessary expenditure for private good;

$y_p - p_n b_n$ = "tolerable tax level".

This is the base to which the state and local tax can be applied. State and local government cannot tax more than $(y_p - p_n b_n)$. $a_n[y_p + G - b_n p_n - \sum_{k=1}^{n-1} (1 - \delta_k)p_k b_k]$ represents the portion of left-over income allocated to private goods. This is the alterable expenditure allocated to private goods.

The denominator $(1 - \delta_k)$ does not appear as the denominator of a_n (as in the case of state and local services 1 to n-1)

because there exists no matching grants for private goods. Equation (3-20) states that tax is equal to *tolerable level of taxes* minus the *alterable expenditure* allocated to private goods.

3.2 Some Theoretical Properties of the Model

It can be seen that equations (3-18) and (3-20) compose a system of n linear equations. The estimation of this system of equations will be discussed in Chapters VI and VII. The theoretical properties of the SALGLERM will be discussed here.

Since the SALGLERM is in fact one form of the Linear Expenditure System (LES), its theoretical properties are the same as the LES. The theoretical properties of the LES can be summarized as follows:[2]

1. The utility function is additively separable. A utility function is said to be additively separable if it has continuous first and second derivatives and if the utility from the consumption of all goods equals the sum of the utility from the consumption of each good. Mathematically a utility function is additively separable if $\frac{\partial U}{\partial X_i}$ and $\frac{\partial^2 U}{\partial X_i^2}$ are continuous and $U(X) = \sum_{i=1}^{n} U(x_i)$.

2. The LES has a built-in economy of parameters. The system uses only 2n-1 independent parameters. These are n-1 of the α's and n of the β's.

3. As noted above, the parameters of the LES has a convenient interpretation.

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[2]For detail discussion of the properties of the LES see Goldberger [12].

4. The LES, however, allows very limited interaction between goods. Every good enters the utility function uniformaly and independently. In this model, goods are almost independent. They cannot be complementary goods. They are, however, substitutes only because they compete for the limited budget. The fact that complementarity is not allowed for in this model can be seen by examining the cross-substitution effect which (this effect) can be obtained by taking partial derivatives of the compensated demand function[3] for good k with respect to p_j.

$$\left[\frac{\partial x_k}{\partial p_j}\right]_{U=\text{const.}} = \frac{a_k \sum_{k=1}^{6} (1-\delta_k)^{a_k} \prod_{k\neq j}^{7} p_k^{a_k} \left(a_j p_j^{a_j - 1}\right)}{(1-\delta_k) p_k \sum_{k=1}^{7} a_k^{a_k}} \qquad (3\text{-}21)$$

It can be seen from equation (3-21) that a negative cross-substitution effect requires some a's to be negative. This is not possible because of the constraint that all a's must be between zero and one. This limitation is, however, not a serious one. The SALGLERM deals with broad categories of goods and services. One does not expect complementarity between broad categories of goods. Complementarity is applied only with narrowly defined goods. For example, one can talk about complementarity between gasoline and automobiles. However, it just does not make much sense to talk about complementarity between highways and education.

[3] For the derivation of the compensated demand function see Appendix A.

CHAPTER IV

Prices

The purpose of this chapter is to generate price data for the model. The role of prices in the model and the rationale for choosing price proxies are discussed. The specific model of the SALGLERM to be estimated, is also specified in this chapter.

4.1 The Specific Model to be Estimated

The SALGLERM has been developed in Chapter III as a general model for the study of state and local government budget decisions. To conduct an empirical study of the state and local government revenue and expenditures using the SALGLERM, the number of goods and services to be included must be specified and described.

Seven goods and service categories are included in this study. They are (1) elementary and high school education; (2) higher education; (3) highways; (4) health and hospitals; (5) police protection; (6) miscellaneous government serivces and (7) private goods. Each of these categories comprise more than 3% of the state and local expenditure. All state and local government expenditure under 3% are combined into the miscellaneous category.

After the included expenditure categories have been specified, the first problem encountered in conducting an empirical study of the SALGLERM is the problem of prices.

4.2 The Role of Prices in the Model and the Choice of Proxies for Prices

The SALGLERM is an allocation model. Prices must enter the model as an allocation device. In this type of model, prices serve as a guideline for the allocation. If prices do not appear in the budget constraint, some notion of cost must appear instead.

For the SALGLERM the term price refers to the same thing as cost because there is no transaction between a seller and a buyer. Thus the market price is not observed. As a result, there is a problem as to what should be used as proxies for the price variables in the model. It should be made clear that the problem of prices is an empirical problem and it is a separate issue from the specification of the SALGLERM. The primary problems here are associated with data and estimation. An attempt must be made to recover price data from available information.

An "actual price", which is defined to be the expenditure divided by the quantity, is not appropriate to use here since quantities are not well measured. The weakness of the measurement of quantity result in errors in price variables. To avoid these errors in variables, the decision is made to use imputed prices as the proxies for price variables. The imputed price is defined as follows:

$$\hat{P}_i = \hat{\zeta}_{io} + \hat{\zeta}_{i1}X_{i1} + \hat{\zeta}_{i2}X_{i2} + \cdots + \hat{\zeta}_{in}X_{in} \qquad (4\text{-}1)$$

Where $\hat{P}_i$ = imputed price of good i

$\hat{\zeta}_{ij}$ $j = 1,\ldots n$ = the least squares estimates of ζ_i's

X_{ij} $j = 1,\ldots n$ = some variable which affects cost of provision of good i.

The use of imputed price is not new. In studying the supply of labor, Robert Hall [13] also used an imputed wage (which is the price of labor, in this case) as an explanatory variable in his model. He argues that the use of imputed wages reduces errors in variables and the resulting estimates are therefore consistent. This procedure is equivalent to an instrumental variable technique.

Imputed prices are used as proxies for prices of elementary and secondary education, higher education, highways, health and hospitals and police protection. Imputed prices are not used for both government miscellaneous services and private goods because these categories consist of several subcategories of goods, which makes it very difficult to derive a well-defined quantity measures for the two total categories. As a result, the decision has been made to use a weighted average of wage as a proxy for the price of government miscellaneous service and a comparative price index as a proxy for the private goods.

4.3 Specification and Estimation of Prices Proxies

In this section specific equations for prices will be specified and estimated.

4.3.1 The price of elementary and secondary education (P_1).-For education, there is a general concensus in using the number of pupils in average daily attendance (ADA) as a quantity measure.[1] An alternative measure, namely, total

[1]See, for example, Riew [14] and Kiesling [15].

enrollment, is considered inferior to the number of pupils in ADA because of the multiple enrollments by the same pupil during the same school year. ADA is not a perfect measure either, because it is affected by differences in absentee rate. Most educators apparently agree with the use of ADA however.[2]

Recall that the term price here is the same as cost. For our purposes here, it should be sufficient to estimate a cost function directly.

Input price variables.-Inputs are labor and capital. For education, labor input is mainly the instructional-staff-labor. Therefore the price of labor should be the average salary of the instructional staff. The use of teacher salaries as an explanatory variable in the cost equation is not unusual [2,16]. The use of wages here is different, however, from Osburn [17] and Riew [14]. Both Osburn and Riew use teacher wages as a quality measure. In their models, the use is justified because they were studying schools in the same area where the difference in wages does not represent the difference in the price of input but rather the difference in the quality of input. (Riew uses schools in Wisconsin as units of observation). However, in the SALGLERM, states across the country are used as the observation and the difference in wages here is used to represent the difference in input price.

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[2]See Kiesling [15].

The price of capital, which is the rate of interest, will be dropped from the cost equation for two reasons. First, capital is assumed to be more mobile than labor and the rate of interest, at a particular time, is assumed to be the same across the country. Secondly, the concern here is with current expenditure, and not the capital outlays. In conclusion, wages alone will be used as input price variables. The assumption is that if wages are high, the cost of ESED should be high. That is, a positive sign of the coefficient of wage is expected.

Scale variables.-A scale variable is included in the cost function because the scale of operation affects unit cost. The number of pupils in ADA per school is widely used as a scale measure [14, 15, 17].

Population density is another alternative. However, the population density measures scale of operation for ESED only indirectly. Therefore the number of pupils in ADA per school is used here and the sign of the coefficient cannot be determined in advance.

Quality variables.-Quality variables are included in the cost function to take care of the variation in qualities. Quality measures can be either input measure or output measures. An output quality is ideally superior to an input measure. This is because the measure of interest is the quality of education, and not the amount or the quality of the inputs of education. However, output quality measures are generally not available.

Kiesling [15] claims that public education is a sector of government where the usable measure of output quality, the achievement score in basic subjects, has existed in refined form for some time. He does not claim, however, that the scores capture the whole of educational quality. The scores, as a measure of output quality of education, have a weakness because differences in scores may be attributable to differences in intelligence and socio-economic background as well as quality of education.

Input quality measures are varied and lengthy but not exhaustive. Percentage of students in high school, number of credit units offered, average number of courses taught per teacher and pupil teacher ratios are examples of input quality measures. There is no way to obtain data for the complete list of input quality measures. Two input quality measures are discussed here.[3]

1. Pupil-teacher ratio (PTR). The smaller the pupil teacher ratio, the higher the "quality" of education. A higher quality of education requires higher cost. Therefore the expected sign of the coefficient of PTR is negative.

2. Percentage of students in high school (PSH). PSH represents the combination of the level of ESED provided by state and local government. A high PSH implies that a great percentage of education is given at the high school level. Of course, high school education costs more than elementary education. Therefore, the expected sign of the coefficient of PSH is positive.

[3]For further discussion of input quality measure see Riew [14].

In conclusion, the price of ESED is specified to be:

$$P_1 = \zeta_{10} + \zeta_{11} WT + \zeta_{12} APPS + \zeta_{13} PTR + \zeta_{14} PSH + \varepsilon_1 \quad (4\text{-}2)$$

where

P_1 = current expenditure per pupil in ADA in public elementary and secondary day school;

WT = average annual salary of the instructional staff (an input price variable);

APPS = average number of pupils in ADA per school (a scale variable);

PTR = pupil-teacher ratio (representing class size, a quality variable); and

PSH = percentage of students in high school (a quality variable representing level combination).

The parameter estimates and t-values (in parentheses) are:

$$P_1 = \underset{(3.012)}{631.390} + \underset{(7.507)}{.096}WT + \underset{(1.400)}{.138}APPS - \underset{(-4.808)}{34.845}PTR + \underset{(.032)}{6.95}PSH$$

$R^2 = .744$, $F(4,43) = 31.2341$

All signs of the coefficients are as expected. The sign of the coefficient of APPS is positive, which implies economies of scale. The coefficient, however, is not significantly different from zero at 95% significant level. Therefore, there is not enough evidence to conclude that economy of scale exists in ESED.

4.3.2 Price of higher education (P_2).-The output measure of higher education (HED) is chosen to be the size of enrollment. This variable, rather than the number of graduates, has been chosen because the education process is considered finished when the course is completed, not when the degree is awarded. Also, choosing this output measure allows one

to discuss the cost of HED per student-year rather than cost of HED per graduate.

Following the ESED, a linear HED cost function is assumed. The HED cost function will include an input price variable, a scale variable and quality variables.

The input price variable is specified to be the average annual salary of the institutional staff of the state and local institutions of higher education (WF), by the same reason that was given for ESED. A positive coeffieicent is expected.

Two input quality measures are: average student faculty ratio (ASFRA) and graduate student to total student ratio (GTR). The first represents a measure of instructor-availability. More students per faculty implies lower quality, and lower quality in turn implies lower cost. Therefore, a negative sign is expected for the coefficient of ASFRA.

The second quality variable measure the level combination and amount of research involved. More research requires more graduate students. More graduate students imply that more of the higher level of education is provided which means better quality and higher cost. Therefore, a positive sign is expected for the coefficient of GTR.

The price equation of HED, therefore, is specified to be

$$P_2 = \zeta_{20} + \zeta_{21}WF + \zeta_{22}ANSPI + \zeta_{23}ASFRA + \zeta_{24}GTR + \varepsilon_2 \quad (4\text{-}3)$$

where

P_2 = current expenditure per student;

WF = average annual salary of the instructional staff of the state and local institutions of higher education;

ANAPI = average number of students per institution;

ASFRA = average student faculty ratio; and

GTR = graduate to total student ratio.

The parameter estimates and the associated t-values are:

$$P_2 = \underset{(5.91)}{2677.24} + \underset{(2.04)}{.0737WF} + \underset{(1.04)}{0.0167ANSPI} - \underset{(-4.32)}{84.23ASFRA} + \underset{(0.10)}{16.53GTR}$$

$$R^2 = 0.3089 \quad F(4,43) = 4.805$$

All coefficients have the expected signs.

4.3.3 Price of highways (P_3).-The highway mileage is chosen as the output measure for highways. The concern here, is with the operating cost, and not with the construction cost. The highway cost equation is specified to be:

$$P_3 = \zeta_{30} + \zeta_{31}WH + \zeta_{32}RMV + \zeta_{33}HAI + \zeta_{34}SNOW + \varepsilon_3 \qquad (4\text{-}4)$$

where

P_3 = price or cost of highway operation per mile;

WH = average wage of highway workers;

RMV = registered motor vehicles;

HAI = highway age index; and

SNOW = the amount of snowfall in the states.

Average wage of highway workers (WH).-Represents the input price variable. A higher wage will give rise to a higher cost. A positive coefficient is expected for this variable.

Registered motor vehicle (RMV).-Represents the extent of the usage of the highway. More vehicles should give rise to more use which in turn will result in greater maintainance costs. A positive coefficient is anticipated for RMV.

Highway age index (HAI).-Is an age variable. This variable is included in the cost function because one would expect an age variable to have an effect on the cost of operation. The highway age index is computed by taking the ratio of the mileage of the old highways to the total milage of highways.[4] The greater the value of HAI the older the highways. Intuitively one would expect a positive sign for the coefficient of HAI since an old road should require higher maintainance and operation cost than a new one. This is, however, not the case since most of the old roads are small roads and they are being abandoned. Fewer people use them and less money is put in for their maintainance and operation. Therefore a negative sign for the coefficient of HAI is expected.

SNOW.-Represents the climate or weather situations of the states. Snow causes trouble in operating and maintaining highways. Heavier snow falls would imply more cost of highway operation. Therefore the sign of the coefficient of SNOW is expected to be positive.

The parameter estimates and the associated t-values are:

$$P_3 = \underset{(1.11)}{1896.09} + \underset{(0.58)}{0.112WH} + \underset{(1.98)}{0.208RMV} - \underset{(-1.40)}{18.00HAI} + \underset{(1.66)}{17.456SNOW}$$

$$P^2 = 0.22 \qquad F(4,43) = 3.05$$

All signs are as expected.

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[4] $HAI = \frac{MORS60}{MORS70} \times 100$ where MORS 60 = miles of roads and streets in 1960. MORS 70 = miles of roads and streets in 1970. The index is greater if greater percentage of roads and streets are constructed before 1960.

4.3.4 Price of health and hospital (P_4).-Health and hospital are classified as one category of state and local government expenditure. The main concern here is in hospital expenditure because it constitutes the main portion of this type of expenditure. An output measure of health and hospital service is specified to be the patient-day equivalent (PDE). The PDE is computed by the following formula.

$$PDE_s = 365ADP_s + \frac{RPOV_s}{RPID_s}(OPV_s) \qquad (4\text{-}5)$$

where

PDE_s = patient-day equivalent for state s;

ADP_s = average daily patient for state s;

$RPOV_s$ = revenue per outpatient visit for state s;

$RPID_s$ = revenue per inpatient day for state s; and

OPV_s = outpatient visits for state s.

Equation (4-5) specifies patient-day equivalent as the sum of two parts. The first part is the patient-day derived from inpatients. $\frac{RPOV(OPV)}{RPID}$ is the inpatient-day equivalent of the outpatient service. With this output measure, the cost equation to be constructed is cost per patient-day equivalent. This cost is assumed to be a linear function of the wage of the hospital personnel, beds per hospital, occupancy rate, average length of stay and personnels per bed. Specifically,

$$P_4 = \zeta_{40} + \zeta_{41}WHH + \zeta_{42}BPH + \zeta_{43}OR + \zeta_{44}ALS + \zeta_{45}PPB + \varepsilon_4 \qquad (4\text{-}6)$$

where

P_4 = expenditure of health and hospital service per patient-day equivalent;

WHH = average annual salary of health and hospital personnel;

BPH = beds per hospital;

OR = occupancy rate;

ALS = average length of stay; and

PPB = personnel per bed.

Average annual salary of health and hospital personnel (WHH) represents the input price variable. As before, a positive coefficient is expected.

Beds per hospital (BPH) is a scale variable. A positive coefficient of this variable would imply a diseconomy of scale and a negative sign would imply an economy of scale.

Occupancy rate (OR) is a measure of the degree of the utilization of fixed inputs. The higher the occupancy rate, the lower the per unit cost. Therefore a negative sign is expected for the coefficient of OR.

Average length of stay (ALS) is the time dimension which affects cost. This variable is included in the cost equation because the output measure selected is the patient-day equivalent which involves a time dimension. One might presume that, the longer the stay, the lower the cost per unit. This is because at the beginning of the admission, the patient needs more treatments, tests and doctors' diagnosis time than in the latter days. When the cost is averaged over the entire length of the stay the per day cost is lower. However, this is just one aspect of the issue. Another aspect is that patients who need a long stay at the hospital are patients with the more serious illnesses and with complications.

According to current medical practices, less severely ill patients are sent home to recuperate as soon as possible. Those who remain in the hospital, in general, require more attention from the personnel and more use of sophisticated equipment. Taking both these considerations into account, a longer stay would imply a higher cost. Thus, a positive sign is expected for the coefficient of ALS.

Personnel per bed (PPB) is a quality variable. The greater the personnel per bed, the greater the cost. Thus a positive sign is expected for this variable. The parameter estimates and the associated t-values are:

$$P_4 = \underset{(1.21)}{1806.11} + \underset{(1.91)}{0.157WHH} + \underset{(0.62)}{0.994BPH} - \underset{(-2.96)}{46.536OR} + \underset{(4.56)}{280.72ALS} - \underset{(-0.62)}{812.9PPB}$$

$R^2 = 0.65$ $\quad F(5,37) = 13.71$

All coefficient of the variables have the expected signs except that of PPB.

4.3.5 Price of police protection (P_5).-The most difficult task in specifying the price equation for police protection is to select an output measure. Several output measures have been used. Hirsch [18], for example, uses population as an output measure in his study about the economy of scale for police protection. Population, however, is not the product of police protection and cannot be a good output measure for police protection.

Schmandt and Stephen [19] use an output index which is the number of activities performed by police departments as

a measure of output. As pointed out by Walzer [20], the index is not a perfect measure of output of police protection because it is affected by the organizational structure of the various police departments. For example, the same service may be provided by two departments with one dividing the service into two distinct functions while the other does not.

Walzer [20] uses an index of service as an output measure of police protection in his study concerning the economies of scales and municipal police services. Walzer's index of service is a composite of the number of offenses cleared, the number of accidents investigated, and the miles traveled by police vehicles. The time spent on each type of activity is transformed to twenty-minute units. Then the index is constructed by summing the frequency of the activity times the average standard unit over all type of activity.

The number of new prisoners which was used as an output measure for police protection in this study has three main weaknesses. First, police activity is more than incarcerating criminals and therefore part of the service provided by police is not represented by the measure. Secondly, part of the process of imprisoning the criminals is out of the police's control. Police may spend time and effort to apprehend a suspected criminal but the individual may not be convicted and imprisoned. Thirdly, there is a time element involved in this measure. A person who is sentenced to jail in 1970 may commit his crime in 1969 and

it is in 1969 that the police service is provided not in 1970. Nevertheless, the number of new prisoners is still counted in 1970. Admittedly the number of new prisoners is not an ideal measure for police service but it is the only choice available with data by states.

The cost equation for police protection is:

$$P_5 = \zeta_{50} + \zeta_{51}WP + \zeta_{52}PPD + \zeta_{53}PPP + \zeta_{54}MVPP + \zeta_{55}TCIPP + \varepsilon_5 \quad (4\text{-}7)$$

where

P_5 = expenditure of police protection per new prisoner;

WP = average annual wage of police personnel;

PPD = number of police per department;

PPP = population per police;

MVPP = state motor vehicle registration per police officer;

TCIPP = total crime index per police.

Average annual wage of police personnel (WP) represents the input price variable. A positive coefficient of WP is expected, of course. The number of police per department (PPD) is a scale variable. This is an input scale, and not an output scale. The sign of the coefficient of this variable cannot be specified in advance.

Population per police (PPP) can be viewed as a measure of quality. The greater the ratio, the lower the quality. Thus a negative coefficient would have been expected. However, greater population per police implies that one policeman has to provide protection for a larger number of people. This prevents him from concentrating on arrests. Also if there are not enough police officers, it is easier for criminals to

escape the arrest. That means the policeman have to put more time and effort in clearing offenses. If this is the case, the expected sign for PPP must be positive.

State motor vehicle registration per police officer (MVPP) is another dimension of quality. More vehicle per police implies lower quality. Since lower quality should result in lower cost, a negative sign is expected for the coefficient of MVPP.

Total crime index per police (TCIP) is the third quality dimension: greater crime index per police means lower quality. Therefore the ratio varies negatively with the cost.

The parameter estimates and the associated t-values are:

$$P_5 = \underset{(-1.24)}{-39728.7} + \underset{(4.79)}{198.33WP} - \underset{(-2.55)}{1350.02PPD} + \underset{(2.07)}{11.78PPP} - \underset{(-2.45)}{23.40MVPP} - \underset{(-0.39)}{43.53TCIP}$$

$R^2 = .56 \qquad F(5,42) = 10.85$

All coefficients have the expected signs. The negative coefficient of PPD indicates that there is an economy of scale in providing police protection. The t-statistic (-2.55) confirms that this coefficient is significantly different from zero at .95 confidence level.

4.3.6 Price of miscellaneous services (P_6).-State and local goods and services which are not classified as one of the five categories above are called "miscellaneous services." Included in miscellaneous services are parks and recreation, financial administration, general control, etc. An explicit output measure for miscellaneous services is difficult to

find because of the aggregative nature of this category. However, one common characteristic of all these services is that they are all labor intensive. This suggests the use of an average wage as a proxy for price. The use of wage as a proxy for price would imply a unit of labor as an output measure. Admittedly, this is not a real output measure but rather an input measure. An annual wage is used because most of the labor contract is made on a year basis. The price of miscellaneous services is calculated from the following formula.

$$P_6 = \sum_{i=1}^{n} \theta_i (MW)_i \qquad (4-8)$$

where

P_6 = price of miscellaneous services;

θ_i = weight given to good or service i.

$$\theta_i = \frac{E_i}{\sum_{i=1}^{n} E_i}$$

Where E_i = expenditure for good or service i which belong to the miscellaneous category;

$(MW)_i$ = wage of workers in service i.

4.6.7 Price of private goods (P_7).-A weighted comparative price index is used as a proxy for the price of private goods. The formula is specified to be:

$$P_7 = \frac{P_m(PP_m)}{100} + \frac{P_n(PP_n)}{100} \qquad (4-9)$$

where

P_7 = weighted average of the index of comparative cost based on an intermediate budget for a four-person family;

P_m = index of comparative cost for metropolitan area;

PP_m = % of population in metropolitan area;

P_n = index of comparative cost for non-metropolitan area;

PP_n = % of population in non-metropolitan area.

The use of a comparative price index as a proxy for price would imply an abstract unit of output measure. Being the only price index in the model, P_7 preserves its relativeness. That is it preserves the relativeness among goods of different categories in the same time period. The reason more than one price index is avoided is because the problem at issue in this model is an allocation problem.

CHAPTER V

Grants

The purpose of this chapter is to generate grant data for the SALGLERM. The role of grants in the model as well as the existing grant structure is first examined. Then, a practical way of separating flat grants and matching grants is formulated. The estimation technique of the grant equations formulated is also discussed in this chapter. Finally the specific grant equation is specified and estimated.

5.1 The Role of Grants in the Model

Previous studies of state and local governments expenditures and revenue ignore the fact that grants of different types play different roles in budget decisions. Henderson [8] for example, lumps all types of grants into his R, which is per capita revenue (of local governments, from federal and state governments). Gramlich [9] puts all federal assistance to state and local governments in his G variable which is defined to be federal grant-in-aids. Horowitz [6] introduces two variables representing grants in her models. One is F_1, the other is F_2. F_1 is defined to be the over-all revenue from the federal government per capita. F_2 refers to per capita grants-in-aid. Horowitz [6, p. 26], however, uses each grant variable in each version of her model. This means that she does not separate flat and matching grants either.

In the SALGLERM, grants are treated differently from previous studies. Total grants are divided into flat grants and matching grants. The flat grant is exogenous to the states and localities. By contrast the matching grant is endogeneous.

A flat grant is independent of the expenditure. A matching grant depends on the amount of state and local government expenditure. A flat grant can be considered as additional income to the state and local governments from which the state and local government are free to spend on anything they want. In effect the flat grant increases the income of state and local governments. Matching grants, in contrast, reduce the price of providing the goods being subsidized. A state or local government will receive a matching grant only when it decides to spend on a certain service for which grants are provided.

Theoretically, the separation allows one to talk about the income effect and price effect of the grants. From the standpoint of econometrics the separation allows one to obtain a real exogeneous G (representing flat grant) in the model.

5.2 The Existing Grant Structure

The distinction between exogeneous and endogeneous grants can be stated mathematically as follows:

Suppose y is the amount of state and local government expenditure, G is a federal grant to state and local government, then the grant G is called an exogeneous grant if $\frac{\partial G}{\partial Y}=0$. It is called an endogeneous grant if $\frac{\partial G}{\partial Y} \neq 0$.

$\frac{\partial G}{\partial Y} = 0$ implies that a change in expenditures does not effect grants. Exogeneous grants are independent of expenditures. Under this definition, the exogeneous grants include block grants, unconditional grants, closed-end matching grants and some open-end matching grants.

Block grants.-A block grant is given to state and local governments as a lump sum. Block grants, whether conditional or unconditional, are not affected by the expenditure at the margin. Therefore, they are classified as exogeneous grant.

Unconditional grants.-Unconditional federal aid to state and local governments is classified as exogeneous since such aid is not affected by expenditures.

Closed-end matching grants.-All closed-end federal matching grants which are fully used are classified as exogeneous grants because they do not affect the decision to spend the marginal expenditure.

Some open-end matching grants.-Open-end matching grants which are given according to some criteria other than the expenditure of any particular goods or services are classified as exogeneous grants. This is due to the fact that the variables with which the grants vary are exogeneous to the systems.

$\frac{\partial G}{\partial Y} \neq 0$ implies that a change in expenditure affects grants. Expenditure is a controllable variable. If grants are affected by expenditures, this portion of grants become endogeneous and the decision-maker can control the size of the grant by varying expenditure.

Conceptually, the structure of grants discussed above can be represented symbolically as follows:

$$G_{xs} = BG_s + UG_s + \sum_{\text{some } p} CEMG_{ps} + \sum_{\text{some } p} OEMG_{ps} \qquad (5\text{-}1)$$

G_{xs} = federal flat (exogeneous) grants to state s;

BG_s = federal block grants to state s;

UG_s = federal unconditional grants to state s;

$CEMG_{ps}$ = closed-end matching grants of program p to state s;

$OEMG_{ps}$ = open-end matching grants of program p to state s.

No further discussion is needed for G_{xs}, BG_s, and UG_s. The last two terms of equation (5-1) need more clarification. A matching grant is a grant from federal to state and local government which requires that state and local government must spend some money to match the fund. A closed-end matching grant is a matching grant which is subject to a certain limitation. In this type of grant the federal government set a maximum amount from which the state and local government can draw the matching fund. Closed end matching grants are classified as flat grants if they are used up to the maximum points. If they are not fully used they must be classified as endogeneous (or matching) grants because the marginal expenditure still affect grants. This is why the summation sign is over some rather than all p. The "some p" is intended to be the grant program which is fully used.

Some of the open-end matching grants are classified as exogeneous grants because the criteria by which the grants vary are exogeneous to the system. For example, a grant for an education program is given according to the number of

students enrolled in elementary school. This is an open-ended grant because state and local government will receive a certain amount of grant for every additional student enrolled. But the number of students enrolled is an exogeneous variable. It is not under the control of the state and local government decision-makers. (Even if birth control programs are effective and under the control of state and local government, the present number of students in primary school is not affected by the present birth control programs.) The last summation sign is intended to sum over the programs in which the amount of grants vary with some criteria other than the level of expenditure of any particular goods or services. Not all the programs which are open end matching grants are classified as endogeneous grants.

The following equation indicates the conceptual frame work of matching grants:

$$G_{esi} = \sum_{\text{some } p} OEMG_{psi} \qquad (5\text{-}2)$$

where

G_{esi} = matching (endogeneous) grans for good i to state s;

$OEMG_{psi}$ = open-end matching grant from program p for good i to state s.

The summation sign represents the sum over some p and signifies the sum of the matching grants from programs of which grants vary with expenditure.

Given eq. (5-1) and (5-2), G and δ can be derived by the following relations

$$G = G_{xs} \qquad (5\text{-}3)$$

$$\delta_i = \frac{\sum_{\text{all } s} G_{esi}}{\sum_{\text{all } s} Y_{si}} \qquad (5\text{-}4)$$

Although the above equations provide a conceptually valid approach, data are not collected in the way that will allow one to utilize equations (5-1) to (5-4) to derive the G and δ's. There is no source of data which gives information about federal grants to state and local government by program and by state. Without these data it is not possible to decompose total grants into matching grants and flat grants. An alternative approach is presented below.

As noted above federal grants to state and local governments can be divided into two parts: endogeneous and exogeneous grants.

$$G_i = G_{ix} + G_{ie} \qquad (5\text{-}5)$$

where

G_i = grant for providing good i;

G_{ix} = the portion of grant which does not vary with expenditure of good i, exogeneous grants; and

G_{ie} = the portion of grant which varies with expenditure of good i, endogeneous grants.

Equation (5-5) states that the grant for good i is equal to exogeneous grants plus endogeneous grants.

$$G_{ix} = \gamma_{io} + \gamma_{i1}D_{i1} + \gamma_{i2}D_{i2} + \ldots\ldots + \gamma_{in}D_{in} \qquad (5\text{-}6)$$

G_{ix} = exogeneous grant for good i;

D_{il}, l=1,...n=some deomgraphic variable which is used as a criterion for grants.

The observational subscript is omitted for simplicity.

Equation (5-6) states that the exogeneous grant for good i is equal to some fixed grant (γ_{oi}) plus some variable grants which vary with D_{i1}, $D_{i2}..D_{in}$ where D_{i1} D_{i2},...D_{in} are demographic variables which are exogeneous to the system. The term γ_{io} represents the portion of exogeneous grants which does not vary with any of the D_{i1}. Three components of grants are incorporated in γ_{io}. They are block grants, unconditional grants and fully used closed-end matching grants. Open-end matching grants which vary with demographic variables are incorporated in $\gamma_{il}D_{il}$, l=1,...n.

$$G_{ie} = \delta_i Y_i \qquad (5\text{-}7)$$

Equation (5-7) states that the endogeneous grant is equal to the federal matching ratio for good i times the expenditure for good i. Substituting equation (5-6) and (5-7) into (5-5), and adding an error term, the equation becomes

$$G_i = \gamma_{io} + \gamma_{il}D_{il} + \gamma_{i2}D_{i2} + \cdots + \gamma_{in}D_{in} + \delta_i Y_i + V_i \qquad (5\text{-}8)$$

where, V_i is a random error term. The needed data which are G and δ_i can be obtained by using

$$G = \sum_{i=1}^{n} \hat{G}_{ix}$$

$$\delta_i = \hat{\delta}_i \qquad (5\text{-}9)$$

Neither equation (5-7) nor equation (5-8) can be estimated because of unavailability of data. There is no data problem in estimating equation (5-8).

5.3 Estimation Problems

There are two major econometric problems associated with equation (5-8). One problem concerns the dependence of Y_i

on G_i; the other pertains to the restriction in the size of δ_i.

Since G_i is the sum of flat grants and matching grants for good i, Y_i is dependent on G_i. That is Y_i is an endogeneous variable. Thus, the ordinary least squares technique will yield an upward biased parameter estimate. One way to get out of this is to use an instrumental variable technique. It is not done here because the parameter of interest is to be estimated with a constraint. This constraint will bring the value of the estimate down. Therefore, an upward bias is not a serious problem at this point.

The second problem involves the restriction in the size of δ_i. Obviously, δ_i can not be greater than one because grants must be less than or equal to expenditure. In fact the maximum value of δ_i can be determined by the following a priori information:

$$0 \leq \delta_i \quad \text{Min} \frac{G_{is}}{Y_{is}} \qquad (5\text{-}10)^1$$

---

1 $G_i = G_{ix} + G_{ie}$ (5-11)

$G_i = G_{ix} + \delta_i Y_i$ (since $G_{ie} = \delta_i Y_i$ from (5-7) (5-12)

That is $G_i \geq \delta_i Y_i$ (since $G_{ix} \geq 0$)

$$\frac{G_i}{Y_i} \geq \delta_i \quad \forall_i \qquad (5\text{-}13)$$

If a second subscript is added to G_i and y_i to indicate the state, then equation (5-13) implies

$$\delta_i \leq \underset{s}{\text{Min}} \frac{G_{is}}{Y_{is}} \qquad (5\text{-}14)$$

The mixed estimation technique was used to estimate equation (5.8) subject to (5-10).[2]

Mixed Estimation Technique[3]

In the mixed estimation technique, $\hat{\delta}_i$ is assumed to be uniformlly distributed over the range between 0 and min $(\frac{G_{is}}{Y_{is}})$. If one supposes that min $(\frac{G_{is}}{Y_{is}}) = k$, then one can write

$$\delta_i = \frac{0 + k}{2} + u = \frac{k}{2} + u \qquad (5\text{-}15)$$

where u has a continuous uniform distribution with mean zero and variance $\frac{(k-0)^2}{12} = \frac{k^2}{12}$. The estimation procedure of the mixed estimation technique is to use the prior information about δ_i in the estimation of all the parameters by using equation (5-15) as an additional observation. Equation (5-15) can be written as

---

The lower bound of δ_i can be derived as follows:

from (5-12) $\quad G_i = G_{ix} + \delta_i Y_i$

When δ_i is minimum, it implies $\delta_i Y_i$ is minimum and that G_{ix} is maximum. The maximum value that G_{ix} can take is G_i. Therefore at minimum δ_i

$G_i = G_i + \delta_i Y_i$,

$0 = \delta_i Y_i$;

$\delta_i = 0$ (since $Y_i \neq 0$).

[2]OLS and a SEARCH techniques were also used to estimate the grant equations but the results were not reported. The OLS estimate of δ is out of the bound of the inequality constraint but the SEARCH estimate of the same parameter is very close to the mixed estimation technique.

[3]For detail development of mixed estimation see Theil and Goldberger [21].

$$\frac{k}{2} = \gamma_o(0) + \gamma_1(0) + \gamma_2(0) + \ldots + \gamma_n(0) + \delta_i(1) + (-u) \qquad (5\text{-}16)$$

The use of equation (5-16) as an additional observation will give rise to heteroschedasticity since the variance of u is not equal to a constant σ^2. The heteroschedasticity can be corrected by multiplying eq. (5-16) by $\frac{\sigma\sqrt{12}}{k}$. After multiplying, eq. (5-16) becomes:

$$\sigma\sqrt{3} = \gamma_o(0) + \gamma_1(0) + \ldots + \gamma_n(0) + \delta_i\left(\frac{\sigma\sqrt{12}}{k}\right) + u^* \qquad (5\text{-}17)$$

where $u^* = \frac{\sigma\sqrt{12}}{k}\ u$

$$\text{Var}(u^*) = \left(\frac{\sigma\sqrt{12}}{k}\right)^2 \ \text{var}(u)$$

$$= \left(\frac{\sigma\sqrt{12}}{k}\right)^2 \ \frac{k^2}{12}$$

$$\left(\because \text{Var}(u) = \frac{k^2}{12}\right)$$

$\therefore$ $\text{Var}(u^*) = \sigma^2$

In practice, the mixed estimator is obtained by adding a new observation to the sample and by using OLS to regress with n + 1 observations, where n is the number of observations in the sample. The new observation is equal to $\sigma\sqrt{3}$ for the left variable, equal to $\frac{\sigma\sqrt{12}}{k}$ for the variable corresponding to δ_i (which is y_i in this case) and equal to zero for the rest of the variables. σ is unknown but can be estimated by $\hat{\sigma}$ which is the sample standard deviation of the sample with n observations.

5.4 Specification of Grant Equations

With the general framework of the grant structure and the estimation technique discussed in the previous sections the next step is to specify the grant equations. The

specifications of grants given below is based on the information contained in the Catalog of Federal Aid to State and Local Government [22], Supplement to the Catalog [23], second supplement to the catalog [24] and State Payment to Local Government [25].

5.4.1 Grants for elementary and secondary education.-

$$PG_1 = \gamma_{10} + \gamma_{11}PP + \gamma_{12}PPL + \gamma_{13}PF + \gamma_{14}PGR + \delta_1 Py_1 + V_1 \quad (5\text{-}18)$$

PG_1 = Federal grants to state and local governments for elementary and secondary schools;

PP = number of pupils;

PPL = number of pupils from low income area;

PF = shared revenue from the lease of federal flood control land;

PGR = annual rent from grazing land leases in force in 1970; and

Py_1 = state and local governments' direct current expenditure on elementary and secondary education.

All variables divided by population to get per capita data.

Federal aid to state and local government for elementary and secondary education is given under several programs. Some of the programs can be classified as exogeneous grants and some as endogeneous grants. Part of the exogeneous grants is reflected in γ_{10}. The other parts varies with PP, PPL, PF, and PGR.

An example of the programs for which grants vary with the number of pupil is School Library Resources, Textbooks, and Other Instructional Materials. To qualify for this grant a state is required to designate an agency to administer the state plan. An acceptable plan must specify criteria for allocating the materials among pupils and teachers of the

state. This grant is distributed to local government on the basis of enrollment [25, p. 21].

Number of pupil from low-income area.-States with more pupils from low-income areas receive more grants for education than states with fewer pupils from low-income areas. This is due to this type of federal aid program which distributed aid to states at a rate of one-half the state average per pupil expenditure for each eligible pupil [25, p. 21].

Shared revenue from the leases of federal flood control lands.-This variable is in the model since part of some grants varies with this revenue. According to the Catalog [22], seventy-five percent of Federal revenue from leasing of lands acquired by the United State for flood control, navigation, and allied purposes, including the development of hydroelectric power, is paid at the end of each fiscal year to the state in which the property is located. This share of the revenue is to be expended, as prescribed by the legislature, for the benefit of public schools and roads of the county or counties in which the land is situated. In most of the states, half of the state's share is distributed for school support [25, p.36]. Matching is not required for this grant.

Annual rent from grazing land leases.-The Federal government shares the federal revenue from grazing fees with the states. The states in turn, redistribute to school districts in which the land is located. The percentage of the share depends on the types of land. The percentage for the grazing districts on public lands is 12.5 percent and for grazing on

lands of land utilization projects is 25 percent. The share is 33.3 percent if the lands are grazing districts on Indian lands ceded to the U.S. for disposition. Finally it is 50 percent if they are leases for grazing on public domain lands outside grazing districts [22 , p. 105; 25 , p. 108].

Y_1 appear in eq. (5-18) to represent endogeneous grants. The following projects are examples of grants for education which can be classified as endogeneous grants.

1. Vocational Education. Federal aid distributed in fixed ratio to local expenditure for approved programs [25, p. 21].
2. Improvement of Science, Mathematics, Foreign Language, and Other Critical Subjects. Federal funds distributed as a fixed ratios of approved local expenditure [25, p. 21].
3. Guidance and Counseling Programs. Federal funds distributed as a fixed ratio of approved local expenditure. [25, p. 31].
4. Adult Basic Education. Federal fund distributed as a fixed ratio of local expenditure [25, p. 21].
5. Work Experience Programs. Federal aids distributed as reimbursement for approved programs [25, p. 21].
6. Manpower development and Training. Federal aids distributed for approved programs [25, p. 23].
7. School Lunch and School Milk Programs. Federal aid distributed as reimbursement of local expenditure [25, p. 23].

8. Neighborhood Youth Corps. Federal aid distributed in fixed ratio to local government expenditure [25, p. 29].

9. Vocational Rehabilitation. Federal aid distributed for approved programs [25, p. 50].

The result of the regressions by mixed estimation technique are as follows:

$$PG_1 = \underset{(-0.57)}{-0.476\ E07} + \underset{(1.72)}{62188.9\ PP} + \underset{(3.34)}{63769.7\ PPL} + \underset{(0.97)}{0.160\ E08\ PF}$$

$$+ \underset{(7.45)}{0.437\ E08\ PGR} + \underset{(2.84)}{0.002\ Py_1} \qquad R^2 = 0.67$$

5.4.2 Grants for higher education.- Federal aid for publicly controlled higher education can be divided into two types. The first type is given by various federal agencies directly to the institutions of higher education. The second type is given to the public higher education institutions through State and local government. The concern of this chapter is with the second type only, because the first type does not go into the state and local government budget. This is not to say that state and local government budget decisions are not affected by the amount of aids colleges and universities received directly from the federal agencies. Rather it is to say that the manner in which the two types of grants enter the model is different. The first type affects the parameters of the model, whereas the second type affects the variables either exogeneous (if it is a flat grant) or endogeneous (if it is a matching grant). Parenthetically, the first type is

given by various federal agencies where as the second type is given by the office of education only. According to the catalog [24, p. 217], federal programs which can be classified as grants for higher education are land-grant colleges, community services and continuing education program, college library assistance and library training and research and improvement of undergraduate instructions.

Land-grant colleges.-Land grant colleges provide a permanent appropriation of a minimum of $150,000 to each state with additional amounts distributed in proportion to population [24, p. 47]. The first part of this grant is clearly a flat grant. The second part is also a flat grant, although it is not easily seen as the first part. This is due to the fact that the number of population in the state is an exogeneous variable to the state and local government.

Community service and continuing education programs.-Under these programs, the commissioner will allot the yearly appropriated sum as follows: (1) $100,000 to each of the 50 states and the District of Columbia; (2) $25,000 each to Guam, American Soma, Puerto Rico, and the Virgin Islands; and (3) the remainder is prorated among the states and other units in proportion to population. This grant can be classified as flat grant for the same reason as given for land grant colleges.

College library assistance and library training and research.-A portion of this grant is fixed, another portion varies with the number of students, still another portion varies with state and local government expenditure [24, p. 103].

Improvement of undergraduate instruction.-This grant is intended to help improve undergraduate instruction in institutions of higher education. Half of the annual appropriation is to be allotted among the states in proportion to the number of students enrolled in institutions of higher education. The other half of each of the appropriation is allotted among the states on the basis of a formula in which the number of students enrolled in institutions of higher education in the state is applied to the allotment ratio for the state [24, p. 116].

From the above discussion the grant equation for higher education can be specified as follows:

$$PG_2 = \gamma_{20} + \gamma_{21} PS_2 + \delta_2 PY_2 + v_2 \qquad (5\text{-}19)$$

where

PG_2 = grant for higher education;

PS_2 = number of students enrolled in publicly controlled institution of higher education in the state;

PY_2 = expenditure for higher education.

The result of the mixed estimation is as follows:

$$PG_2 = 279729 + 16.32\ PS_2 + 0.004\ PY_2 \qquad R^2 = 0.23$$
$$(1.22) \quad (2.71) \quad (2.03)$$

5.4.3 Grants for highways.-Federal grants to state and local governments which are used for highways come through several programs. An investigation of the Census of Governments [25] and Highway Statistics [26] indicates that federal grants which are used for highways come from the following programs.

1. The Federal Highway Trust Fund.-This fund which is administered by the Federal Highways Administration, provides grant-in-aids for use in highway improvements. These grants are allotted to States in accordance with formulas that give weight to population and mileage.

2. Federal Forest Reserve Revenue.-Twenty-five percent of the federal revenue from national forests is returned to the state of origin. The states in turn distribute part of these funds to counties in which such forests are located, for roads.

3. Federal Flood Control Revenue.-Of federal revenue from leasing of land acquired by the federal government for flood control purposes, 75 percent is returned to the states in which lands are located. The state's share is in turn distributed to counties in which the projects are located, for roads.

4. Federal Grazing Revenue.-Part of the federal grazing revenue arising within the state is returned to the state. The state then redistributes it to the counties of origin for schools and roads.

Given the above information, the equation for grants for highways is specified to be the following:

$$PG_3 = \gamma_{30} + \gamma_{31}PMORS70 + \gamma_{32}PA + \gamma_{33}PGR + \gamma_{34}PF + \delta_3 PY_3 + V_3 \qquad (5\text{-}20)$$

where

PG_3 = federal grants for highways;

PMORS70 = miles of roads and streets in 1970;

PA = area of the forest;

PGR = annual rental of grazing land leases in force in 1970;

PF = share revenue from the leases of lands acquired by the federal government for flood control purposes; and

PY_3 = state and local current highway expenditure.

The results of the regression by mixed estimation technique are as follows:

$$PG_3 = \underset{(2.97)}{0.450\ E07} + \underset{(2.7)}{73.65\ PMORS70} + \underset{(1.17)}{185.76PA} + \underset{(1.90)}{0.151\ E08\ PGR} - \underset{(-0.74)}{0.149PF} + \underset{(2.27)}{0.087PY_3} \qquad R^2 = 0.62$$

5.4.4 Grants for health and hospitals.-A study of grants for health and hospital lead to the conclusion that all grants of this type are matching grants [25, Table 7]. Examples of the programs are listed below.

1. Local health services. Federal aid is distributed in fixed ratio to local expenditures for federally aided public health activities [25, p. 27].

2. Mental health services. Federal aid is distributed in fixed ratio to local expenditure for approved costs [25, p. 71].

3. Crippled children's services. Federal aid distributed in fixed ratio to approved local expenditure [25].

4. County health units. Federal aids distributed in fixed ratio to local expenditure for federally aided public health activities [25, p. 47].

5. Health facilities planning. Federal aids distributed in fixed ratio to local expenditure.

The federal matching ratio for health and hospital can be computed by using the following relation.

$$G_{4s} = \delta_4 Y_{4s} \qquad (5\text{-}21)$$

$$\delta_4 = \frac{G_{4s}}{Y_{4s}}$$

$$\hat{\delta}_4 = \text{Min } (\frac{G_{4s}}{Y_{4s}})^4$$

The value of $\hat{\delta}_4$ was computed to be .047.

5.4.5 Grants for police protection.-The law enforcement assistance Act of 1965 authorizes the Attorney General to make grants to state and local government for law enforcement purposes. The grants can be given in the form of training grants, project grants and technical assistance. The training grants is for programs which trains state and local law enforcement personnel. The project grant is for projects which are designed to promote or demonstrate improved method of law enforcement. The technical assistance authorized the Attorney General to cooperate with and render technical assistance to state and local governments in such matters as law enforcement organization, techniques, and practices, and the prevention and control of crime.

All law enforcement assistance can be classified as matching grants. The federal matching ratio for law enforcement assistance can be computed from the following relation.

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[4]Using min δ_4 as $\hat{\delta}_4$, may under-estimate matching grant, but this is the only way to avoid negative flat grant.

$$G_{5s} = \delta_5 Y_{5s} \qquad (5\text{-}22)$$

where

G_{5s} = grant for police protections to state s;

Y_{5s} = expenditure for police protection.

$$\delta_5 = \frac{G_{5s}}{Y_{5s}}$$

$$\hat{\delta}_5 = \text{Min } (\frac{G_{5s}}{Y_{5s}})$$

The value of $\hat{\delta}_5$ was computed to be .002.

5.4.6 Grants for miscellaneous goods.-Most of the grants for miscellaneous goods are endogeneous grants. Recall that the concern here is for grants which are used for current expenditure. All programs which provide grants for miscellaneous goods but not for current expenditures, for example grants for airport construction, will not be included here. There are many programs that can be classified as grants for miscellaneous goods. The programs listed below are the programs which appeared in the 1967 Census of Government [25].

Libraries: Federal funds distributed in fixed ratio to local expenditure for library service and construction.

Diaster Relief: Federal funds distributed on the basis of need.

Outdoor Recreation: Federal funds distributed in fixed ratio to local expenditure for acquiring and developing facilities.

Work experience program: Federal funds distributed as reimbursement for approved programs.

Open-Space Land: Federal funds distributed in fixed ratio to local expenditure for acquisition, development and preservation of open space land.

Civil Defense: Federal funds distributed on basis of need or as fixed ratio to local expenditure.

Cuban Refugee Aid: Federal funds distributed as reimbursement of local expenditure.

Urban Planning Assistance: Federal funds distributed for approved projects.

Economic Opportunity Aid: Federal fund distributed in fixed ratio to local expenditure for approved programs.

Mass Transit: Federal funds distributed in fixed ratio to local expenditure for study, analysis, planning, and development.

An investigation of these grant programs shows that all of the grants are endogeneous grants. Even the diaster relief and the civil defense can be considered endogeneous. These two programs are provided on the basis of need and not on the basis of a proportion of the expenditure. However, these grants must be closely related to expenditure of diaster relief and civil defense. In other words, the implicit criterion is the amount of expenditure spent on these two items. Although all the programs listed have been considered endogeneous grants, it is believed that there is a large portion of this type of grants (grants for miscellaneous goods) which may be classified as exogeneous. This latter portion of grants for miscellaneous goods may arise from programs

which are not listed above. The decision is, therefore, made to regress grants for miscellaneous goods. The regression equation is specified to be:

$$PG_6 = \gamma_{60} + \delta_6 PY_6 + V_6 \qquad (5\text{-}23)^5$$

where

$$PG_6 = G_6/POP.; \quad PY_6 = Y_6/POP.$$

γ_{60} is supposed to catch all the portion of grants which do not vary with expenditure. Data for G_6 are not readily available. Therefore, data were generated by using the relation:

$$G_6 = GP(R) - \sum_{i=1}^{5} G_i$$

where

G_6 = Grants for miscellaneous goods;

GP = Intergovernmental revenue from Federal government to state and local government;

R = ratio of direct current expenditure to total expenditure;

$R = \frac{CE}{TE}$; and

G_i = grants for good i. i=1,...5.

[5]The estimate of this equation will be biased.

CHAPTER VI

Estimation Procedures

The purpose of this chapter is to present the theory of the estimation of the linear expenditure system (LES) in general and the SALGLERM in particular. First, the general problems of the estimation of the linear expenditure system will be outlined. Then the available procedures for the estimation of the linear expenditure system will be examined. One estimation procedure will be selected to estimate the SALGLERM and finally the application of the selected procedure to the SALGLERM will be discussed.

6.1 The General Problems in the Estimation of the LES

For statistical purposes the linear expenditure system (LES) can be written in the following form:

$$y_i = \beta_i p_i + \alpha_i \left(m - \sum_{j=1}^{n} \beta_j p_j\right) + u_i \qquad (6\text{-}1)$$

$$i = 1,\ldots n$$

y_i = a T x 1 vector of observation of expenditures on the i^{th} good;

β_i = a parameter to be estimated, a scalar, i = 1,...n

p_i = a T x 1 vector of observation of prices of the i^{th} good;

α_i = a parameter to be estimated, a scalar;

m = a T x 1 vector of observation of the total expenditure (income);

u_i = a T x 1 vector of unobserved random disturbance.

In estimating the LES one encounters the following estimation problems.

1. Nonlinearity. The linear expenditure model is linear in the variables but not linear in the parameters. In the equations to be estimated (eq. 6-1) the coefficient of p_k is $\alpha_k\beta_k$ which is nonlinear.

2. Singularity of the covariance matrix. The full system of n equations (6-1) necessarily has a singlar covariance matrix since $\sum_{i=1}^{n} u_i = 0$. This can be proved as follows:

From (6-1)

$$y_i = \beta_i p_i + \alpha_i (m - \sum_{j=1}^{n} \beta_j p_j) + u_i$$

$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \beta_i p_i + m \sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n} \alpha_i \sum_{j=1}^{n} \beta_j p_j + \sum_{i=1}^{n} u_i$$

$$\because \sum_{i=1}^{n} \alpha_i = 1 \text{ and } \sum_{i=1}^{n} y_i = m$$

$$m = \sum_{i=1}^{n} \beta_i p_i + m - \sum_{j=1}^{n} \beta_j p_j + \sum_{i=1}^{n} u_i$$

$$\Rightarrow \sum_{i=1}^{n} u_i = 0$$

3. Uniqueness of the estimators. The β_i's i=1,...n appear in every equation. Estimating each equation one by one, will not guarantee that the same estimated value for the β's will be obtained.

The LES can be estimated by four different procedures:

1. Stone's Procedure (SP).

2. Modified Stone's Procedure (MSP).

3. Pollak and Wales' Procedure (PWP).

4. Parks' Procedure (PP).

Each procedure will be examined briefly and then the application of the Parks' Procedure to the SALGLERM will be discussed.

6.2 Stone's Procedure (SP)

Given the value of α_i equation (6-1) can be written as:

$$(y_i - \alpha_i m) = [-\alpha_i p_1 \ldots (1-\alpha_i) p_i - \alpha_i p_{i+1} \ldots - \alpha_i p_n] \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} + u_i \quad (6\text{-}2)$$

or $$Y_i = X_i \beta + u_i \quad (6\text{-}3)$$

where $Y_i = y_i - \alpha_i m$

$X_i = [-\alpha_i p_1 \ldots (1 - \alpha_i)\ p_i - \alpha_i p_{i+1} \ldots - \alpha_i p_n]$

β = an n x 1 vector of β_i's

The complete system of n equations can be written as

$$Y^* = X^* \beta + U^* \quad (6\text{-}4)$$

where $Y^* = \begin{pmatrix} Y_1 \\ \vdots \\ Y_n \end{pmatrix}$; $X^* = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}$; and

Tn x 1 T n x n

$U^* = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}$ $\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix}$

T n x 1 n x 1

Then given the value of β_i equation (6-1) can be written as:

$$Y_i - \beta_i P_i = \alpha_i \left(m - \sum_{j=1}^{n} \beta_j P_j\right) + u_i \qquad (6\text{-}5)$$

$$(i = 1,\ldots,n)$$

This equation follows directly from (6-1)

or $W_i = \alpha_i Z + u_i \qquad (6\text{-}5)$

where $W_i = (y_i - \beta_i P_i) \qquad (6\text{-}6)$

$$Z = \left(m - \sum_{j=1}^{n} \beta_j P_j\right)$$

In this case, the complete system of n equations can be written as

$$W^* = Z^{*\prime}\ \alpha + U^* \qquad (6\text{-}7)$$

where $W^* = \begin{pmatrix} W_1 \\ \cdot \\ \cdot \\ \cdot \\ W_n \end{pmatrix}_{Tn \times 1}$; $Z^{*\prime} = \begin{pmatrix} Z & & 0 \\ & Z & \\ 0 & & Z \end{pmatrix}_{Tn \times n}$;

$\alpha = \begin{pmatrix} \alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_n \end{pmatrix}_{n \times 1}$; $U^* = \begin{pmatrix} u_1 \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{pmatrix}_{Tn \times 1}$

The Stone's estimation procedure involves the following steps [27]:

(1) assume a set of initial value of α_i's

(2) apply OLS to (6-4) obtaining $\hat{\beta}_j$

(3) using $\hat{\beta}_j$ from (2), apply OLS to (6-6), getting $\hat{\alpha}_i$

(4) go to step (2) using $\hat{\alpha}_i$

(5) the iteration procedure continues until some measure of convergence is reached.

6.3 Modified Stone's Procedure (MSP)

The Stone's procedure ignores the fact that the covariance of u_i is singular. This is because $\sum_{i=1}^{n} u_i = 0$.

Thus, one of the equations in (6-1) is redundant in the sense that it can be uniquely determined by using n - 1 equations. That is, y_n can be expressed as a linear combination of the n-1 equations. It is dealt here with allocation problems, once the n - 1 commodities are allocated, the n^{th} commodity is the residual.

Because of the interdependence among the u's, OLS will be inefficient. In the Modified Stone's Procedure (MSP), the n^{th} equation is deleted and an alternative iteration scheme is used to arrive at the MLE estimators. After deleting the n^{th} equation from the system, (6-4) can be written as

$$Y = X\beta + U \qquad (6\text{-}8)$$

where $Y = \begin{pmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_{n-1} \end{pmatrix}_{T(n-1) \times 1}$ $\quad X = \begin{pmatrix} X_1 \\ \cdot \\ \cdot \\ X_{n-1} \end{pmatrix}_{T(n-1) \times 1}$

and $U = \begin{pmatrix} u_1 \\ \cdot \\ \cdot \\ u_{n-1} \end{pmatrix}_{T(n-1) \times 1}$

Equation (6-7) can be written as

$$W = Z^{*}\alpha + U \qquad (6\text{-}9)$$

where

$$W = \begin{bmatrix} W_1 \\ \vdots \\ W_{n-1} \end{bmatrix}_{T(n-1) \times 1} \quad ; \quad Z^* = \begin{bmatrix} Z & & 0 \\ & Z & \\ 0 & & Z \end{bmatrix}_{T(n-1) \times (n-1)}$$

$$\alpha = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_{n-1} \end{bmatrix}_{(n-1) \times 1} \qquad U = \begin{bmatrix} u_i \\ \vdots \\ n-1 \end{bmatrix}_{T(n-1) \times 1}$$

Stochastic Specification

(1) $u_{it} \sim N(0,\Sigma) \qquad i=1,\ldots,n-1$

That is, the random disturbances $(u_{1t}, u_{2t},\ldots u_{n-1,t})$ for observation t are assumed to come from a multivariate normal distrubution with a mean zero and covariance matrix Σ.

(2) $E(u_{it}, u_{j(t+s)}) = 0 \qquad s \neq 0 \quad \forall_{i,j}$

That is, the disturbances for different observations are assumed to be uncorrelated.

(3) $E(U) = 0$

The expected value of the random disturbance vector is equal to zero.

(4) $E(UU') = \Omega = \Sigma \otimes I$

where

Σ = (n-1)(n-1) covariance matrix of the multivariate normal disturbance.

$$\Sigma = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1,n-1} \\ \vdots & & \vdots \\ \sigma_{n-1,1} & \cdots & \sigma_{n-1,n-1} \end{pmatrix}$$

Σ is assumed to be positive definite and symmetric.

$I = T \times T$ identity matrix

$\otimes$ = direct product (or Kronecker-product)

$$\Omega = \Sigma \otimes I = \begin{pmatrix} \sigma_{11} I & \cdots & \sigma_{1,n-1} I \\ \vdots & & \vdots \\ \sigma_{n-1,1} I & \cdots & \sigma_{n-1,n-1} I \end{pmatrix}_{(n-1)T \times (n-1)T}$$

The full system of n equations (6-4) or (6-7) necessarily has a singular covariance matrix. This is because $\sum_{i=1}^{n} u_i = 0$. Previous estimation procedures ignored this fact. Stone's procedure incorrectly assumes that the covariance matrix is a scalar matrix $\sigma^2 I_{nT \times nT}$. Malinvaud [28, p. 310-14] allows a more general covariance matrix, but still does not recognize the singularity.

The joint density function of u can be written as

$$f(u|\Sigma) = (2\pi)^{-(n-1)T/2} |\Sigma|^{-T/2} \exp\{-1/2\, u'(\Sigma^{-1} \otimes I)u\}\, du$$

Transforming from the u's to y's gives, from (6-8)

$$f(y|\Sigma,\alpha,\beta) = k|\Sigma|^{-T/2} \exp\{-1/2(Y - X\beta)'(\Sigma^{-1} \otimes I)(Y - X\beta)\}dy|J|^{T} \qquad (6\text{-}10)$$

or from (6-9)

$$= k|\Sigma|^{-T/2} \exp.\ \{-1/2(W-Z^*\alpha)'(\Sigma^{-1}\otimes I)(W-Z^*\alpha)\}dy|J|^T \qquad (6\text{-}11)$$

The Jacobian $|J| = 1$ and k is a constant of proportionality. Taking the log of (6-10), yields

$$L = k' + \frac{T}{2}\log|\Sigma^{-1}| - \frac{1}{2}(Y - X\beta)'(\Sigma^{-1}\otimes I)(Y - X\beta) \qquad (6\text{-}12)$$

or taking log of (6-11) yields,

$$L = k' + \frac{T}{2}\log|\Sigma^{-1}| - \frac{1}{2}(W - Z^*\alpha)'(\Sigma^{-1}\otimes I)(W-Z^*\alpha) \qquad (6\text{-}13)$$

Let σ^{ij} = an element of the inverse of the covariance matrix Σ^{-1}. Taking partial derivative of L with respect to σ^{ij} and equate $\frac{\partial L}{\partial \sigma^{ij}}$ to zero, yields

$$s_{ij} = 1/T(W_i - Z\alpha_i)'(W_j - Z\alpha_j)$$

or (6-14)

$$s_{ij} = 1/T(Y_i - X_i\beta)'(Y_j - X_j\beta)$$

as estimator of σ_{ij}.

Let S be the ML estimator of Σ. Substitute S back in (6-12), the likelihood function can be written as

$$L^* = k^* + T/2 \log|S^{-1}| \qquad (6\text{-}15)$$

Differentiating (6-15) with respect to α_h, a typical element of α, we have

$$\frac{\partial L^*}{\partial \alpha_h} = \frac{T}{2}\frac{\partial}{\partial \alpha_h}\log|S|$$

$$= -\frac{T}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\frac{\partial \log|S|}{\partial s_{ij}}\cdot\frac{\partial s_{ij}}{\partial \alpha_h}$$

$$= \sum_{i=1}^{n-1} s^{ih}Z'(W_i - Z\alpha_i) \qquad (6\text{-}16)$$

$$i = 1 \qquad h = 1,2,\ldots.(n-1)$$

Differentiating (6-15) partially with respect to the vector β yields,

$$\frac{\partial L^*}{\partial \beta} = -\frac{T}{2}\frac{\partial}{\partial \beta}\log|S|$$

$$= -\frac{T}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\frac{\partial \log|S|}{\partial s_{ij}}\frac{\partial s_{ij}}{\partial \beta}$$

$$= \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} s^{ij}[X_i'Y_j - X_i'X_j\beta] \qquad (6\text{-}17)$$

The ML estimators of α and β are obtained by setting (6-16) and (6-17) equal to zero and solved for β and α. Since this system is nonlinear in the parameters (because of the involvement of α and β in X, Y, W and Z) it is necessary to resort to numerical methods for the solutions. Equating (6-16) to zero, a set of least square normal equations is obtained. These equations can be solved for α_i given the value of β which appears in W_i and Z.

This estimator can be shown to be the simple least squares estimator; the weight, s^{ih} does not have any effect. Equating (6-17) to zero, the generalized least squares normal equations are obtained. These equations can be solved for β given the values of α, which is involved in Y_i and X_i.

In summary, the modified Stone procedure involves the following steps:

1. Select a set of initial values of β and use OLS to estimate α_i by using $\alpha_i = (Z'Z)^{-1}Z'W_i$
2. Estimate S by using $s_{ij} = \frac{1}{T}(W_i - Z\alpha_i)'(W_j - A\alpha_j)$
3. Inverse S from step (2)
4. Estimate β by using full information GLS $\beta = (X'S^{-1}X)^{-1}X'S^{-1}Y$.

5. Go to step (1) using β from step (4).
6. The iteration proceeds until some measure of convergence is reached.

Comment on this procedure

1. The modified Stone procedure (MSP) does not produce a valid covariance matrix for the estimators. At each round, the procedure gives a covariance matrix of $\hat{\alpha}$, given the value of β or vice versa. This obviously ignores the covariance between the two estimators.
2. In step 2, a GLS estimate is needed. In order to be able to apply GLS, the covariance matrix must be known. In general, we do not know the covariance matrix, so GLS cannot be obtained. What is obtained by using S as the estimate of the covariance matrix is the Feasible Aitken Estimator.
3. The selection of the n^{th} equation to be deleted is arbitrary. The values of the estimates depend on which equation is deleted.

6.4 Pollak and Wales Procedure (PWP)

Pollak and Wales' [29] stochastic specification differs from MSP. They assume that the disturbance terms are the errors in the β_i's and the covariance matrix of the error terms is a multiple of $\hat{x}^2_{it}$. The maximum likelihood estimates are obtained by dropping one equation out of the full system. With this new stochastic specification, Pollak and Wales prove that the estimators do not depend on which equation is dropped.

6.5 Parks' Procedures (PP)

Parks [30] uses the same stochastic specification as MSP. The differences between PP and MSP is that PP uses the information matrix and Gauss-Newton procedure to replace MSP's iteration method. Parks procedure also gives estimates of the covariance matrix of the MLE's.

To obtain the maximum likelihood estimate of the linear expenditure model, one must solve the equations

$$\begin{pmatrix} \frac{\partial L^*}{\partial \alpha} \\ \\ \frac{\partial L^*}{\partial \beta} \end{pmatrix} = 0 \qquad (6\text{-}18)$$

The MSP's iteration method is one way of solving equation (6-18). An alternative procedure, based on Gauss-Newton numerical method, suggested by Parks is to compute the second derivative of the likelihood function with respect to the parameters.

$$M = \begin{bmatrix} \frac{\partial^2 L^*}{\partial \alpha \partial \alpha'} & \frac{\partial^2 L^*}{\partial \alpha \partial \beta'} \\ \\ \frac{\partial^2 L^*}{\partial \beta \partial \alpha'} & \frac{\partial^2 L^*}{\partial \beta \partial \beta'} \end{bmatrix} \qquad (6\text{-}19)$$

This matrix M is used to find the miximum likelihood estimates of α and β based on a Taylor series expansion. The values of the parameters on step i + 1 is computed from the following equations.

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix}_{i+1} = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}_{i} + \lambda_i \, [-M_i]^{-1} \cdot G_1 \qquad (6\text{-}20)$$

where

M_i = the Hessian Matrix defined in (6-19) evaluated at $\alpha=\alpha^i$, $\beta=\beta^i$. The superscript refers to the iteration number.

λ_i = a scalar which is chosen to maximize the likelihood function

G_i = the gradient vector evaluated at $\alpha=\alpha^i$, $\beta=\beta^i$.

The matrix M is also used to compute the estimates of the covariance matrix of maximum likelihood estimates of α and β. The consistent estimates of the covariance matrix is

$$\left[-M \right]^{-1}_{\substack{\alpha=\hat{\alpha}\\ \beta=\hat{\beta}}} \qquad (6\text{-}21)$$

6.6 Comparison of the Four Estimation Procedures

1. Stochastic Specification

In SP, MSP and PP the random variable u_i comes in additively as the disturbance term of equation i. SP assumes that $u \sim N(0, \sigma^2 I)$. MSP and PP assume that $u \sim N(0,\Sigma)$.

In PWP, the random variable u_i comes in as the error in β_i and $u_i \sim N(0, D_i)$.

2. Properties of the Estimators

a) In SP, the properties of the estimators are unknown

b) In MSP, the estimators are

1) MLE
2) no covariance matrix for the estimators is given
3) depending on which equation is deleted
4) given the deleted equation the estimators for the β_i's are unique

c) In PWP, the estimators are

1) MLE
2) not depending on which equation is chosen as residual
3) the covariance matrix for the estimators is provided.

d) In PP, the estimators are

1) MLE
2) the covariance matrix of the estimators is given
3) given the deleted equation, the estimators for the β_i's are unique

After careful considerations of the properties of the estimates, the decision was made to use PP to estimate the SALGLERM. This decision was based on the attractive properties of the estimates of PP and also on the fact that Professor Parks kindly made his program available to me.[1] The application of PP to the SALGLERM is the subject of the next section.

6.7 The Application of PP to the SALGLERM

The expenditure equations (equations (3-18)) and the tax equation (equation (3-20)) of the SALGLERM can be rearranged and redefined in order to fit in the frame work of the LES. From (3-18)

$$(1-\delta_k)p_k x_k = (1-\delta_k)p_k b_k + a_k[y_p + G - b_7 p_7 - \sum_{j=1}^{6}(1-\delta_j)p_j b_j] \quad (6\text{-}22)$$
$$k=1,\ldots.6$$

After adding a disturbance term, equation (6-22) can be rewritten as

$$y_k = \beta_k p_k + \alpha_k[m - \sum_{j=1}^{7} p_j \beta_j] + u_k \quad (6\text{-}23)$$
$$k = 1,\ldots 6$$

where $y_k = (1-\delta_k)p_k x_k$

$$\beta_k = (1-\delta_k)b_k \qquad k=1,\ldots 7$$

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[1] MAXLES, the Maximum Likelihood Estimation for the Linear Expenditure System.

$$\delta_7 = 0$$

$$\alpha_k = a_k$$

$$m = y_p + G$$

From (3-20)

$$y_p - T = p_7 b_7 + a_7 [y_p + G - b_7 p_7 - \sum_{j=1}^{6} (1-\delta_j) p_j b_j] \qquad (6\text{-}24)$$

After adding the error term, equation (6-24) can be rewritten as

$$y_7 = \beta_7 p_7 + \alpha_7 [m - \sum_{j=1}^{7} p_j \beta_j] + u_7 \qquad (6\text{-}25)$$

where

$$y_7 = (1-\delta_7)(y_p - T)$$

$$\alpha_7 = a_7$$

$$\beta_k = (1-\delta_k) b_k$$

$$\alpha_7 = 0$$

Equation (6-23) and (6-25) can be expressed as one equation as

$$y_k = \beta_k p_k + \alpha_k [m - \sum_{j=1}^{7} p_j \beta_j] + u_k \qquad (6\text{-}26)$$

$$k=1,\ldots 7$$

PP can be applied to equation (6-26) directly. The MAXLES program is then used to estimate the SALGLERM.

CHAPTER VII

Empirical Results and Policy Implications

7.1 Empirical Results

The SALGLERM has been estimated with the Parks' procedure. The prices used in the estimation are the imputed prices as estimated in chapter IV.[1] Equation 6 which relates to the expenditure for other goods and services is treated as a residual. The results are reported in Table 1.

All the signs of the coefficients are as expected, except the sign of β_7. The negative sign of β_7, although not expected, is theoretically possible. It implies that an increase in the price of private goods would result in an increase in state and local taxes. This can be shown by taking partial derivative of the tax equation (eq. 3-20) with respect to p_7. Since $\beta_k = (1 - \delta_k)b_k$, a negative β_k would imply a negative b_k.

$$\frac{dT}{dp_7} = -b_7 + a_7 b_7 = (a_7 - 1)b_7$$

Since $a_7 - 1 < 0$, and b_7 is negative, therefore $\frac{dT}{dp_7}$ is positive. The increase in taxes can be explained by the fact that an increase in the price of private goods make the private goods relatively more expensive than publicly provided goods. Then it is more efficient to shift expenditures from private goods to public goods.

[1] The SALGLERM has also been estimated with actual prices. The results are similar and are not reported here.

Table 1. Estimated Value of the Parameters of the SALGLERM*

Commodity	α value	β value
1. Elementary and High school Education	0.04656	16.147699
	(0.00632)	(6.803167)
2. Higher Education	0.01658	7.699620
	(0.00245)	(1.971044)
3. Highways	0.00690	5.625651
	(0.00102)	(1.262572)
4. Health and Hospital	0.00983	-0.445786
	(0.00143)	(1.573868)
5. Police Protection	0.00461	0.022252
	(0.00068)	(0.018875)
6. Miscellaneous Services	0.02931	1.066713
	(0.00441)	(1.073685)
7. Private Goods and Services	0.88617	-0.009184
	(0.01537)	(0.510644)

*The values in the parenthesis are the standard errors.

β_4 is negative implying that the system is operating on the elastic portion of the demand curve for health and hospital. The fact that a negative value of β_k implies that the demand for good k is elastic can be shown as follows:

Taking the partial derivative of the expenditure equation for good k (eq. 3-18) with respect to p_k yields,

$$p_k \frac{\partial x_k}{\partial p_k} + x_k = b_k - a_k b_k$$

$$\frac{p_k}{x_k} \frac{\partial x_k}{\partial p_k} + 1 = \frac{b_k}{x_k} (1 - a_k)$$

$$\frac{p_k}{x_k} \frac{\partial x_k}{\partial p_k} = -1 + \frac{b_k}{x_k} (1 - a_k)$$

$$\eta_k = -1 + \frac{b_k}{x_k} (1 - a_k)$$

Since $x_k > 0$, $0 < a_k < 1$, if b_k is negative, the $|\eta_k|$ is greater than 1. This means that the demand for good k is elastic when b_k is negative.

The fact that b_k appears in every equation in the system as a coefficient of price k tempts one to conclude that if b_k is negative, good k must be a substitute of good j. This is not the case, however. A close look at the system will indicate that the estimated equations are expenditures and tax equations. They are not demand equations. Complementarity and substitution should be defined on the basis of cross-substitution effect of the Slutsky equation.[2] Substitute

[2]For a duscussion about complementarity and substitution goods and Slutsky equations see Henderson and Quandt [31] p. 37.

goods have a positive cross-substitution effect, whereas complementary goods have a negative cross-substitution effect. The cross-substitution effect is the derivative of the compensated demand function. The compensated demand function for the SALGLERM is[3]

$$x_k = \frac{a_k Z \prod_{k=1}^{6} (1 - \delta_k)^{a_k} \prod_{k=1}^{7} p_k^{a_k}}{(1 - \delta_k) p_k \prod_{k=1}^{7} a_k^{a_k}} + b_k$$

$$\left(\frac{\partial x_k}{\partial p_j}\right)_{U=\text{const.}} = \frac{a_k Z \prod_{k=1}^{6} (1 - \delta_k)^{a_k} \prod_{k=1, k\neq j}^{7} p_k^{a_k}}{(1 - \delta_k) p_k \prod_{k=1}^{7} a_k^{a_k}} (a_j p_j^{a_j - 1}) \quad (7\text{-}1)$$

It can be seen from equation (7-1) that the cros-substitution effect is free from b. Therefore, the sign of b has no effect whatever on the complementarity or substitutability nature of the good.

A negative value of b is not unexpected. Parks, [32, p. 644] Pollak and Wales [29, pp. 613 and 621] and Phlips [33, p. 455] all obtain some negative values of b's in their estimations of the linear expenditure systems.

The estimates of α can be interpreted as the percent of supernumerary income devoted to each function. That is 4.66, 1.66, 0.69, 0.98, 0.46, 2.93 and 88.61 percent of the

[3]See Appendix A for a detailed derivation of the compensated demand function.

supernumerary income is allocated to elementary and high school education, higher education, highways, health and hospital service, police protection, miscellaneous services and private goods and services respectively.

Care must be taken in interpreting the estimated value of β. One should recall that β_k is different from b_k. Actually $b_k = \frac{\beta_k}{1-\delta_k}$ the value of β_k, δ_k and the corresponding value of b_k are tabulated in Table 2.

Table 2. b Values

k	β_k	δ_k	$1-\delta_k$	b_k
1	16.148	0.002	0.998	16.180
2	7.699	0.004	0.996	7.700
3	5.625	0.087	0.913	6.161
4	-0.445	0.047	0.953	-0.466
5	0.022	0.002	0.998	0.022
6	1.066	0.324	0.676	1.577
7	-0.009	0.000	1.000	-0.009

The value of b can be interpreted as the quantity of goods and services which does not depend on prices. This interpretation[4] is based on the fact that b appears as the constant term of the demand function.[5] With this interpretation one can say that in a society of one thousand people[6]

[4]This interpretation is an extrapolation beyond the range of the observation. It is given here because an interpretation is needed.

[5]See equation (3-17). [6]The equation is estimated on the basis of a thousand population.

16 people will be educated up to high school level regardless of the cost. Eight people will be given higher education, six miles of road will be provided and about two public employees will be hired to take care of miscellaneous services. All these will be done regardless of the cost. There is no amount of private goods and services which is not sensitive to price. According to the data used, there is not enough evidence to indicate that the amount of health and hospital service, which will be provided by state and local governments irrespective of cost, is different than zero.

The Significance of the Model

The purpose of this section is to test the overall performance of the model. Since the model is nonlinear in the parameters and the method of estimation used is a full information maximum likelihood technique, R^2 is not applicable. Since the estimators are estimated by the process of maximizing a likelihood function the appropriate test is a likelihood ratio test. The null hypothesis to be tested is

$$H_o: \alpha_i = \beta_j = 0 \qquad i = 1,....7 \text{ and } i \neq 6 \qquad j = 1,....7.$$

The test statistic to be used is

$$\lambda = \frac{\text{Max } L_o}{\text{Max } L} \qquad (7\text{-}2)$$

Where L_o is the value of the likelihood function under H_o. L is the value of the likelihood function without the

constraint of the hypothesis. It has been shown[7] that

$$-2\ell n\lambda \sim \chi^2(r) \qquad (7\text{-}3)$$

Where r is the number of constraints in the hypothesis. The null hypothesis is rejected if $-2\ell n\lambda > \chi^2(r)$. In practice, if the program gives the value of log likelihood function at each iteration, one only has to obtain the difference between the value of the log likelihood function when all the parameters are equal to zero and the value of the log-likelihood function when the parameters are at the optimum.

The log-likelihood function of the SALGLERM can be written as

$$L^* = k^* + \frac{T}{2} \ln |S^{-1}| \qquad (7\text{-}4)^8$$

The MAXLES program minimizes $\frac{T}{2} \ln |S|$. Since the MAXLES give the value of $\frac{T}{2} \ln |S|$ for each iteration, $\ln \lambda$ can be obtained.

Using this log likelihood ratio test, the null hypothesis is rejected at .01 level. The degree of freedom used is r = 14. The reason r is equal to 14 in spite of the fact that only 13 values of the parameters are specified is because the 14th parameter is uniquely determined by the 13 parameters. The specification of 13 parameters implies that all 14 parameters are specified.

It should be noted that the null hypothesis tested does not imply that all the parameters are equal to zero. Under the null hypothesis all the β_i are zeroes, but only 6 of

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[7] See S.S. Wilks [34, p. 408-411], S.M. Goldfeld and R.E. Quandt [35, p. 72-74].

[8] See eq. (6-15), p. 69.

the α_i's are zeroes. By specifying 6 α's to be zero, the implication is that the residual α is equal to 1. That is α_6 is equal to 1. The rejection of the null hypothesis implies that there is evidence that the full model performs better than a model with $\alpha_6 = 1$ and the rest of the parameters equal to zero. It is known, however, that the performance of a model with $\alpha_6 = 1$, the rest of the parameters equal zero is at least as good as when there is no explanatory variable at all. Therefore, it can be concluded that the performance of the SALGLERM is significantly better than when there is no explanatory variable at all.

7.2 Policy Implications

In this section we will discuss the effects of various changes in the parameters or the exogeneous variables on the values of the endogeneous variables. This will yield additional information to the policy makers when a decision has to be made as to what policy action has to be taken.

The Effect of a Change in the Rate of Matching Grant on Expenditures and Taxes

The effect of a change in the rate of a matching grant on expenditures and taxes can be obtained by taking partial derivatives of expenditures and taxes with respect to the rate of matching grants. The expenditure equations are

$$Y_k = p_k b_k + \frac{a_k}{1-\delta_k} \left[Y_p + G - b_7 p_7 - \sum_{k=1}^{6} (1-\delta_k) p_k b_k \right] \qquad (7\text{-}5)$$

$$k=1,\ldots.6$$

The tax equation is

$$T = Y_p - p_7 b_7 - a_7 \left[Y_p - b_7 p_7 + G - \sum_{k=1}^{6} (1-\delta_k)\, p_k b_k \right] \qquad (7\text{-}6)$$

$$\frac{\partial y_k}{\partial \delta_k} = a_k[y_p + G - b_7 p_7 - \sum_{\substack{j=1 \\ j\neq k}}^{6} (1-\delta_j) p_j b_j]/(1-\delta_k)^2 \qquad (7\text{-}7)$$

Since both the numerator and denominator are positive, $\frac{\partial y_k}{\partial \delta_k}$ is positive. This indicates that an increase in the rate of matching grants of service k would result in an increase in the expenditure of service k. The effect of the change in the matching rate on expenditure for each state can be quantitatively determined by substituting in the values of the exogeneous variables in equation (7-7).

The effect of a change in the matching rate for good j on the expenditure of good k can be obtained by taking the partial derivative of the expenditure equation for good k with respect to δ_j.

$$\frac{\partial y_k}{\partial \delta_j} = \frac{a_k}{1-\delta_j} p_k b_k$$

The sign of $\frac{\partial y_k}{\partial \delta_j}$ depends on the sign of b_k. If b_k is positive, then $\frac{\partial y_k}{\partial \delta_j}$ is positive. If b_k is negative $\frac{\partial y_k}{\partial \delta_j}$ is negative. If $\frac{\partial y_k}{\partial \delta_j}$ is positive then it implies that an increase in the expenditure for good j will result in an increase in the expenditure on good k. This is the case if more matching grants are given to good j, then less of the state and local government's own money is needed for good j and more can be spend for good k.

If $\frac{\partial y_k}{\partial \delta_j}$ is negative the implication is that an increase in the rate of a matching grant for good j will give rise to a decrease in the expenditure on good k. This can happen

when the demand for good k is elastic. The increase in the matching rate for good j induces the state and local governments to spend more on good j and then are left with less revenue to spend on good k.

The effect of a change in the matching rate on taxes can be obtained by taking the partial derivative of the tax equation with respect to δ_k.

$$\frac{\partial T}{\partial \delta_k} = -a_7 b_k p_k$$

This implies that an increase in the matching rate for good k would result in a decrease in taxation. It should be recalled that this is state and local taxes while the matching rate is the rate of grant given by the federal government. If a larger grant is given to good k less state and local tax should be needed.

The Effect of a Change in Per Capita Income on Expenditure and Tax

$$\frac{\partial y_k}{\partial y_p} = \frac{a_k}{1-\delta_k}$$

This means that an increase in per capita income will induce an increase in the expenditure.

$$\frac{\partial T}{\partial y_p} = 1-a_7$$

This is also positive and implies that an increase in income would result in an increase in the state and local taxes. The sensitivity of expenditure and tax with respect to income is given in Table 3.

The Effect of a Change in the Flat Grant on Expenditure and Tax

$$\frac{\partial y_k}{\partial G} = \frac{a_k}{1-\delta_k} > 0$$

$$\frac{\partial T}{\partial G} = -a_7 < 0$$

This implies that an increase in a flat grant increases expenditure but reduces the taxes. The sensitivity of expenditure and taxes with respect to flat grants is given in Table 3.

Table 3. The Sensitivity of Expenditure and Tax with Respect to Income and Flat Grants

k	$\partial y_k/\partial y_p$ or $\partial T/\partial y_p$	$\frac{\partial y_k}{\partial G}$ or $\frac{\partial T}{\partial G}$
1	.047	.047
2	.017	.017
3	.008	.008
4	.011	.011
5	.005	.005
6	.043	.043
T	.114	-.886

It can be seen from Table 3 that the effect of a change in flat grants on expenditures is the same as the effect of a change in per capita income on expenditures. The effect of a change in per capita income on taxes and the effect of a change in flat grants on taxes are different, however. An increase of a dollar in personal income will result in an

increase of $0.114 in state and local taxes. An increase of a dollar in federal flat grants, however, will decrease the state and local taxes by $0.886.

It should be noted that $\frac{\partial T}{\partial y_p} - \frac{\partial T}{\partial G} = 1$. This is not surprising because it follows from the specification of the model. The model, however, does not specify the magnitude of a_7 because that is determined empirically. Therefore the empirical results of the model show that the effect of a reduction in the federal income tax and the effect of an increase in flat grants on state and local taxes are different in both sign and magnitude.

The value of $\frac{\partial y_k}{\partial y_p}$ and $\frac{\partial y_k}{\partial G}$, which are equal, can be interpreted that a dollar increase in either personal income or flat grants will result in $.047, .017, .008, .011, .005, .043 increase in expenditures for primary and secondary education, higher education, highway services, health and hospital services, police protections and miscellaneous services respectively.

The Effect of the Changes in Prices on Expenditure and Taxes

$$\frac{\partial y_k}{\partial p_k} = b_k - a_k b_k = (1-a_k)b_k$$

$$k = 1,.....6$$

This partial derivative is positive if b_k is positive. It is negative if b_k is negative. A positive $\frac{\partial y_k}{\partial p_k}$ implies that the demand for good k is inelastic. A negative $\frac{\partial y_k}{\partial p_k}$ implies that the demand for good k is elastic. The value of $\frac{\partial y_k}{\partial p_k}$ is tabulated in Table 4.

$$\frac{\partial Y_k}{\partial p_j} = \frac{-a_k}{1-\delta_k}\ (1-\delta_j)b_j$$

$$k=1,....6$$
$$j=1,....6$$
$$j\neq k$$

The cross effect of price on expenditure is negative if b_j is positive and vise versa. The sign of $\frac{\partial Y_k}{\partial p_j}$ indicates the direction of the change in the expenditure for good k given the direction of the change in the price of good j. This sign, however, has nothing to do with the substitutability or complementarity of good k and j.[9]

$$\frac{\partial T}{\partial p_k} = a_7(1-\delta_k)b_k$$

$$k=1,....6$$

The sign of $\frac{\partial T}{\partial p_k}$ will be positive or negative depending on the sign of b_k. If $\frac{\partial T}{\partial p_k}$ is positive, the implication is that an increase in the price of good k will result in an increase in taxes. If $\frac{\partial T}{\partial p_k}$ is negative the opposite is true. The value of $\frac{\partial T}{\partial p_k}$ is given in Table 4.

$$\frac{\partial T}{\partial p_7} = -b_7 + a_7 b_7 = (a_7-1)b_7$$

This is positive since b_7 is negative and a_7-1 is negative. This means that an increase in the price of the private good would result in an increase in the taxes. This is because the price of private goods are high relative to the price of publicaly provided goods, then it pays to consume more state and local goods. The value of $\frac{\partial T}{\partial p_7}$ is given in Table 4.

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[9] See page 79, Chapter VII.

Table 4. The Effect of Prices on Expenditures and Taxes

P \ Y	1	2	3	4	5	6	T
1	15.420	-0.257	-.129	-0.178	-0.080	-0.694	14.308
2	-0.036	7.569	-.061	-0.084	-0.038	-0.330	6.794
3	-0.264	-0.095	6.118	-0.062	-0.028	-0.242	4.983
4	0.020	0.007	0.003	-0.461	-0.002	0.019	-0.393
5	-0.001	-0.000	-.000	-0.000	0.022	-0.000	0.018
6	-0.050	-0.018	-.008	-0.011	-0.005	1.531	0.944
7	0.000	0.000	0.000	0.000	0.000	0.000	0.001

In Table 4, the values along the diagonal are the own price effects. The values off the diagonal are the cross price effects. The values in the last column represents the change in taxes with respect to the change in prices. The negative value of $\frac{\partial T}{\partial P_4}$ is the result of a negative b_4 and should be disregarded. The bottom row of Table 4 consists of almost all zeroes which imply that a change in the price of private goods has no effect on the expenditure for state and local goods and services. Also the figures in row 5 indicate that the change in the cost of police protection has very little effect on the expenditure on other services. The figures in the first row imply that the change in the cost of education tends to have greater effects on other expenditures. This may be explained by the fact that education constitutes a larger percentage of total expenditure than other categories.

The Effect of the Changes in Demographic Variables

A change in the demographic variables can affect expenditure and tax through prices and grants. For example, the effect of a change in the salary of elementary and high school teachers on expenditure and taxes can be computed as follows:

$$\frac{\partial p_1}{\partial WT} = \zeta_{11}$$

$$\frac{\partial y_1}{\partial WT} = \frac{\partial y_1}{\partial p_1} \cdot \frac{\partial p_1}{\partial WT} = (1-\alpha_1) b_1 \zeta_{11} = 1.486$$

$$\frac{\partial T}{\partial WT} = \frac{\partial T}{\partial p_1} \frac{\partial p_1}{\partial WT} = \alpha_7 (1-\delta_1) b_1 \zeta_{11} = 1.373$$

The effect on the expenditure and tax of the changes in other variables which affect prices and grants could be computed in the same way.

CHAPTER VIII

Suggestions for Further Research

It has been demonstrated, in the last chapter, that the SALGLERM is capable not only of predicting the direct effects of changes in flat grants, matching grants, per capita income, prices or costs of provision of goods and services on state and local expenditures and taxes, but also of predicting the indirect effects on such expenditures and taxes due to changes in the demographic variables which affect either prices or grants. The model not only gives the effects on the expenditures as a whole as most models do, but even gives the effect on each expenditure category. Although it is true that this model is capable of answering more policy questions than any single model built for this purpose, it is also true that there are several policy questions which are not answered by this model.

Certainly, to develop a more sophisticated model to answer more questions, more research is needed. To serve as a guide line for future research the possible expansion of the model is discussed.

Possible Expansion of the Model

1. The SALGLERM as it now stands does not include investment expenditure in the model. In reality, state and local governments do have investment expenditures. They also

finance the investment expenditure by debt. Further research could be done by introducing investment expenditure and debt into the model. In such a case the cost of provision of goods and services would involve not only price but also interest. The trade off would be no longer between current goods and services only but between current goods and services and future goods and services as well. Interest and prices must interact in that model.

There are two possible ways of introducing investment into the model. One is to treat investment as another good. The other is to divide each type of expenditure into two parts: capital expenditures and current expenditures. In the second method more information will be obtained but also more degrees of freedom will be lost. In the first case, however, less information is obtained but also less degrees of freedom are lost.

2. The SALGLERM assumes that all states have the same population parameters. That is, all the a's and b's are constant across the states. This implies that all the states have the same utility function with the same parameters. This also means that no differences in tastes, politics and other noneconomic institutions are allowed to affect budget decisions. In other words the differences among states allowed are in the variables, not in the strucutres. This is not realistic, since different states may have different tastes which will give rise to different a_i's and b_i's. One could modify the SALGLERM so that it incorporates differences

in tastes, politics and other noneconomic institutions. This could be done by writing b_i and a_i as a function of taste, politics and other noneconomic variables. This will, of curse, complicate the model both theoretically and empirically. More research needs to be done to develop an acceptable estimation technique which will handle these complications.

3. The short-coming of the SALGLERM discussed in the last paragraph is inherent when the model is used with cross-section data. When the SALGLERM is used with the time series data another restriction became obvious. The SALGLERM does not allow its parameters to change over time. The SALGLERM is a static model. A dynamic version of the SALGLERM can be developed by allowing the parameters to vary with time. The functional form of the parameters can be either stochastic or nonstochastic. Of course, if stochastic functional form is specified, then a random coefficient regression technique must be used.[1]

4. Still another version of the SALGLERM can be developed by combining cross-section and time-series data. This model could incorporate taste and politics as well as time variables in the parameters. The model can be a dynamic model with random coefficients and variable parameters.

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[1] For Random Coefficient Regression see Langham and Mara [36], for Dynamic Version of Linear Expenditure Model see Louis Phlips [33].

APPENDIX A

The Compensated Demand Function

The compensated demand function for the SALGLERM can be derived by minimizing the budget equation subject to the utility constraint.

$$\text{Min } G = \sum_{k=1}^{6} (1-\delta_k)\ p_k x_k - T$$

$$\text{Subject to } U^0 = \sum_{k=1}^{6} a_k \ln(x_k - b_k) + a_7 \ln\left(\frac{y_p - T}{p_7} - b_7\right)$$

A Lagrangean function can be set up as follows.

$$L = \sum_{k=1}^{6} (1-\delta_k) p_k x_k - T + \mu\left(U^0 - \sum_{k=1}^{6} a_k \ln(x_k - b_k) - a_7 \ln\left(\frac{y_p - T}{p_7} - b_7\right)\right)$$

$$\frac{\partial L}{\partial x_k} = (1-\delta_k)\ p_k - \mu \frac{a_k}{x_k - b_k} = 0 \qquad k=1,\ldots,6 \qquad (1)$$

$$\frac{\partial L}{\partial T} = -1 - \mu \frac{a_7}{\frac{y_p - T}{p_7} - b_7}\left(-\frac{1}{p_7}\right) = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \mu} = U^0 - \sum_{k=1}^{6} a_k \ln(x_k - b_k) - a_7 \ln\left(\frac{y_p - T}{p_7}\right) - b_7) = 0 \qquad (3)$$

From (1)

$$(1-\delta_k)\ p_k = \frac{\mu a_k}{x_k - b_k} \qquad (3)'$$

$$a_k = \frac{(1-\delta_k)\ p_k\ (x_k - b_k)}{\mu} \qquad k=1,\ldots,6 \qquad (4)$$

$$\mu = \frac{(1-\delta_k)\ p_k\ (x_k - b_k)}{a_k} \qquad k=1,\ldots,6 \qquad (5)$$

From (2)

$$\frac{\mu a_7}{\frac{y_p - T}{p_y} - b_7} \left(\frac{1}{p_7}\right) = 1$$

$$a_7 = \frac{p_7}{\mu}\left(\frac{y_p - T}{p_7} - b_7\right) \tag{6}$$

or $$\mu = \frac{p_7}{a_7}\left(\frac{y_p - T}{p_7} - b_7\right) \tag{7}$$

Raising (5) to a_kth power,

$$\mu^{a_k} = \frac{(1-\delta_k)^{a_k}\, p_k^{a_k}\, (x_k - b_k)^{a_k}}{a_k^{a_k}} \qquad k=1,.....6 \tag{8}$$

Raising (7) to a_7th power,

$$\mu^{a_7} = \frac{p_7^{a_7}}{a_7^{a_7}}\left(\frac{y_p - T}{p_7} - b_7\right)^{a_7} \tag{9}$$

Multiplying the six equations of (8) and equation (9) yields,

$$\mu^{\sum_{k=1}^{7} a_k} = \frac{1}{\pi_{k=1}^{7} a_k^{a_k}} \prod_{k=1}^{6} (1-\delta_k)^{a_k} \prod_{k=1}^{7} p_k^{a_k} \prod_{k=1}^{6} (x_k - b_k)^{a_k} \left(\frac{y_p - T}{p_7} - b_7\right)^{a_7} \tag{10}$$

Taking the natural logs of equation (10), yields,

$$\sum_{k=1}^{7} a_k \ln \mu = -\sum_{k=1}^{7} a_k \ln a_k + \sum_{k=1}^{6} a_k \ln(1-\delta_k) + \sum_{k=1}^{7} a_k \ln p_k$$

$$+ \sum_{k=1}^{6} a_k \ln (x_k - b_k) + a_7 \ln \left(\frac{y_p - T}{p_7} - b_7\right)$$

Since $\sum_{k=1}^{7} a_k = 1$, by assumption,

$$\ln \mu = -\sum_{k=1}^{7} a_k \ln a_k + \sum_{k=1}^{6} a_k \ln (1-\delta_k) + \sum_{k=1}^{7} a_k \ln p_k + \{ \sum_{k=1}^{6} a_k \ln(x_k-b_k) + a_7 \ln(\frac{y_p-T}{p_7} - b_7) \}$$

The term in the brackets is equal to U° by eq. (3), therefore

$$\ln \mu = -\sum_{k=1}^{7} a_k \ln a_k + \sum_{k=1}^{6} a_k \ln(1-\delta_k) + \sum_{k=1}^{7} a_k \ln p_k + U° \qquad (11)$$

From (3)'

$$x_k - b_k = \frac{a_k}{(1-\delta_k)p_k} \mu$$

$$\ln (x_k-b_k) = \ln(\frac{a_k}{(1-\delta_k)p_k}) + \ln \mu \qquad (12)$$

Substituting the value of ln μ from (11) into (12) yields,

$$\ln (x_k-b_k) = \ln a_k - \ln (1-\delta_k) - \ln p_k - \sum_{k=1}^{7} a_k \ln a_k + \sum_{k=1}^{6} a_k \ln (1-\delta_k) + \sum_{k=1}^{7} a_k \ln p_k + U°$$

$$k=1,....6 \qquad (13)$$

From (7)

$$(\frac{y_p-T}{p_7} - b_7) = \frac{a_7}{p_7} \mu$$

$$\ln(\frac{y_p-T}{p_7} - b_7) = \ln \frac{a_7}{p_7} + \ln \mu \qquad (14)$$

Substituting the value of μ from (11) into (14), yields,

$$\ln (\frac{y_p-T}{p_7} - b_7) = \ln a_7 - \ln p_7 - \sum_{k=1}^{7} a_k \ln a_k + \sum_{k=1}^{6} a_k \ln (1-\delta_k) + \sum_{k=1}^{7} a_k \ln p_k + U° \qquad (15)$$

Equations (13) and (15) are the logarithmic form of the implicit compensated demand functions for the SALGLERM.

This implicit demand function can be expressed explicitly in the form of x_k as a function of the P's as follows:

Define Z such that $\ln Z = e^{u^\circ}$. Thus, equation (13) can be written as:

$$\ln (x_k - b_k) = \ln a_k - \ln (1-\delta_k) - \ln p_k - \sum_{k=1}^{7} a_k \ln a_k + \sum_{k=1}^{6} a_k \ln (1-\delta_k) + \sum_{k=1}^{7} a_k \ln p_k + \ln Z.$$

Then,

$$\ln (x_k - b_k) = \ln \frac{a_k \prod_{k=1}^{6} (1-\delta_k)^{a_k} \prod_{k=1}^{7} p_k^{a_k} Z}{(1-\delta_k) p_k \prod_{k=1}^{7} a_k^{a_k}}$$

$$x_k - b_k = \frac{a_k \prod_{k=1}^{6} (1-\delta_k)^{a_k} \prod_{k=1}^{7} p_k^{a_k} Z}{(1-\delta_k) p_k \prod_{k=1}^{7} a_k^{a_k}}$$

$$x_k = \frac{a_k \prod_{k=1}^{6} (1-\delta_k)^{a_k} \prod_{k=1}^{7} p_k^{a_k} Z}{(1-\delta_k) p_k \prod_{k=1}^{7} a_k^{a_k}} + b_k \qquad (16)$$

$$k=1,\ldots.6$$

Equation (16) is the compensated demand function for good k.

Equation (15) can be rewritten as:

$$\ln\left(\frac{Y_p - T}{p_7} - b_7\right) = \ln a_7 - \ln p_7 - \sum_{k=1}^{7} a_k \ln a_k$$

$$+ \sum_{k=1}^{6} a_k \ln (1-\delta_k) + \sum_{k=1}^{7} a_k \ln p_k + \ln z.$$

$$\ln\left(\frac{Y_p - T}{p_7} - b_7\right) = \ln \frac{a_7 \prod_{k=1}^{6} (1-\delta_k)^{a_k} \prod_{k=1}^{7} p_k^{a_k} z}{p_7 \prod_{k=1}^{7} a_k^{a_k}}$$

$$\frac{Y_p - T}{p_7} - b_7 = \frac{a_7 z \prod_{k=1}^{6} (1-\delta_k)^{a_k} \prod_{k=1}^{7} p_k^{a_k}}{p_7 \prod_{k=1}^{7} a_k^{a_k}}$$

$$T = Y_p - \left(\frac{a_7 z \prod_{k=1}^{6} (1-\delta_k)^{a_k} \prod_{k=1}^{7} p_k^{a_k}}{p_7 \prod_{k=1}^{7} a_k^{a_k}} + b_7\right) p_7$$

$$T = Y_p - \frac{a_7 z \prod_{k=1}^{6} (1-\delta_k)^{a_k} \prod_{k=1}^{7} p_k^{a_k}}{\prod_{k=1}^{7} a_k^{a_k}} - b_7 p_7 \qquad (17)$$

Equation (17) is the counterpart of the compensated demand function for tax. Whereas eq. (16) gives the relationship between the quantity of the state and local goods and prices, given that utility is held constant, eq. (17) gives the relationship between state and local taxes and the prices given that utility is fixed.

The own substitution effect can be computed from eq. (16) as follows:

$$\left(\frac{\partial x_k}{\partial p_k}\right)_{U=Const.} = \frac{a_k Z \prod_{k=1}^{6}(1-\delta_k)^{a_k}}{(1-\delta_k)\prod_{k=1}^{7} a_k^{a_k}} \prod_{\substack{j=1 \\ j\neq k}}^{7} p_j^{a_j} \left((a_k-1)\, p_k^{a_k-2}\right) \quad (18)$$

Since $a_k>0$, $Z>0$, $\prod_{k=1}^{6}(1-\delta_k)^{a_k}>0$, $(1-\delta_k)>0$, $\prod_{k=1}^{7} a_k^{a_k}>0$,

$\prod_{\substack{j=1 \\ j\neq k}}^{7} p_j^{a_j}>0$, $a_k-1<0$, $p_k^{a_k-2}>0$.

$$\therefore \left(\frac{\partial x_k}{\partial p_k}\right)_{U=const.} < 0$$

The own substitution effect is always negative.

For taxes, from (17)

$$\frac{\partial T}{\partial p_7} = \frac{-a_7 Z \prod_{k=1}^{6}(1-\delta_k)^{a_k}}{\prod_{k=1}^{7} a_k^{a_k}} \prod_{k=1}^{6} p_k^{a_k} \left(a_7 p_7^{a_7-1}\right) - b_7$$

$$\left(\frac{\partial T}{\partial p_7}\right)_{U=Const.} = \frac{-a_7^2 Z \prod_{k=1}^{6}(1-\delta_k)^{a_k} \prod_{k=1}^{6} p_k^{a_k} p_7^{a_7-1}}{\prod_{k=1}^{7} a_k^{a_k}} - b_7 \quad (19)$$

Since $a_7^2>0$, $Z>0$, $\prod_{k=1}^{6}(1-\delta_k)^{a_k}>0$, $\prod_{k=1}^{6} p_k^{a_k}>0$, $p_7^{a_7-1}>0$,
$\prod_{k=1}^{7} a_k^{a_k}>0$, therefore the first term of eq. (19) is positive.

Let the first term of Eq. (19) be T_1. The,

$$\left(\frac{\partial T}{\partial p_7}\right)_{U=const.} = -T_1-b_7$$

Since b_7 can be positive or negative, the sign of $\frac{\partial T}{\partial p_7}$ can be either positive or negative.

If b_7 is positive, $\frac{\partial T}{\partial p_7}$ is negative.

If b_7 is negative, then the sign of $\frac{\partial T}{\partial p_7}$ depends on the comparative size of T_1 and b_7. If $|b_7| > T_1|$, then $\frac{\partial T}{\partial p_7} > 0$. If $|b_7| < |T_1|$, then $\frac{\partial T}{\partial p_7} < 0$. If $|b_7| = |T_1|$, and the sign of b_7 is different from the sign of T_1, then $\frac{\partial T}{\partial p_7} = 0$.

If $\left[\frac{\partial T}{\partial p_7}\right]_{U=\text{const.}}$ is positive, then it implies that to keep utility constant, an increase in price of private goods must be accompanied by an increase in tax. If $\frac{\partial T}{\partial p_7}$ is negative then an increase in the price of private goods must be accompanied by a decrease in tax to keep utility constant.

APPENDIX B

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BIOGRAPHICAL SKETCH

Somchai Richupan was born December 5, 1938, in Angtong, Thailand. He was graduated with a Bachelor of Education from Chulalongkorn University, Bangkok, Thailand, in 1962, and was graduated with a Bachelor of Economics from the University of Thammasat, Bangkok, Thailand in 1964. He taught high school in Thailand from 1964-1970.

Mr. Richupan came to the United States in June 1970. He began his graduate study in economics at the University of Florida in September, 1970 and he received his Masters of Arts degree in Economics in December 1971.

Somchai Richupan is married to the former Sumol Rajatawan and is a father of three daughters, Aujit, Imchai and Ngamjit. He is a member of Phi Kappa Phi.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

F. A. Sloan, Chairman
Associate Professor of Economics

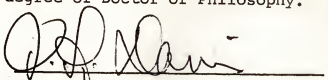
I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

M. R. Langham
Professor of Food and Resource Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

R. Lusky
Assistant Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



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This dissertation was submitted to the Graduate Faculty of the Department of Economics in the College of Business Administration and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1974

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